Physics Module 7

Electricity and Magnetism

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I. Electricity and Magnetism

By Dr. Sam Kinyera Obwoya (Kyambogo University Uganda)

II. Prerequisite Course or Knowledge

As a prerequisite to study this module, you need a background of high school physics; basic concepts of differential and integral calculus and vector methods. It might be a good idea to refresh your knowledge, if you feel that your knowledge of calculus and vector methods is inadequate then you need to consult any Mathematics book on calculus and vector analysis. However, you don’t have to despair as most of the content will be treated very simply that you may have no problem in following.

III. Time

The time recommended for you to complete this course is 120 hours

IV. Materials

- Internet Connection
- Compulsory Readings And Compulsory Resources (As Listed In Sections 11 & 12)
- Software Related To This Module
V. Module Rationale

This unit is designed to provide experiences for the student that will lead him/her into an understanding of the similarities and differences among electric, magnetic, and gravitational fields. The inquiry projects used here will support instruction in electrical circuits, gravitational dynamics, and electromagnetic phenomena of all sorts.

Electricity and magnetism forms a core component of Physics that one needs in understanding some other components of physics like atomic physics, solid state physics, where these ideas can aid in the understanding of such fundamental electric phenomena as electric conductivity in metals and semi conductors. It is hoped that this module will give clear perception of what physics is really about that is so needed for life in the world today especially in the teaching of school physics.

VI. Overview

This course of Electricity and magnetism is intended for students enrolling for B.Ed Degree. The module consists of five units: Concept of electric charge; electric potential; capacitance; direct current and magnetism. The study of electric charge involves differentiating between conductors and insulators and using them to demonstrate the existence of charges. In addition, Coulomb’s law will be stated and its expression derived and used in calculations. Along with this, electric field, dipole moments; potential energy; and torque on an electric dipole and flux of electric field will be defined. Their expressions will be derived and also used to solve problems.

Under electric potentials, the sub-topics will be handled and relevant expressions shall be derived and used for calculations. In the third section of the module, capacitance, properties of capacitors, including capacitors with dielectric will be learnt. For the section on Direct current and circuits, derivation of microscopic form of Ohm’s law will be among the expressions to be derived. Also analysis of equivalent circuits will be dealt with. Finally Magnetism will form the last part of the module of which Ampere’s curitual law will form part of it.
6.1 Outline

Unit 1: Electric charge (20 hours)
- Conductors & Insulators.
- Coulomb's Law.
- Electric Field (E).
  - \( \vec{E} \) due to a point charge.
  - \( \vec{E} \) due to electric dipole, line of charge, charged disk.
  - Dipole in an electric field;
  - Potential energy torque of an electric dipole.
  - A charged isolated conductor.

Unit 2: Flux of an Electric Field (10 hours)
- Gauss law:
- Gauss's law and Coulomb's law
- A charged isolated conductor
  - Cylindrical symmetry,
  - Planar symmetry,
  - Spherical symmetry.

Unit 3: Electric Potential (V) (15 hours)
- Equipotential surfaces. \( V = V(E) \).
- \( V \) due to
  - a point charge,
  - electric dipole,
  - a continuous distribution.

- \( \vec{E} = \vec{E} \ (V) \) due to isolated conductor.
- Van de Graaff accelerator.

Unit 4: Capacitance (C) (15 hours)
- Calculating the capacitance:
  - a parallel-plate capacitor
  - cylindrical capacitor,
  - a spherical capacitor,
- Capacitors in parallel and in series.
• Storing energy in an electric field.
• Capacitors with dielectric.

Unit 5: Direct Current (30 hours)
• Basic Concepts. The Schematic Diagram Kirchoff's Laws.
• Resistivity. Equations with Multiple unknowns
• Mesh Analysis Equivalent Circuits Maximum Power Transfer.
• Power Transfer Efficiency

Unit 6: Magnetism: (30 hours)
• Magnetic field, magnetic flux, flux and density.
• The magnetic force on a current-carrying wire.
• Moving charge in a magnetic field.
• The Oscilloscope. Faraday's law and electromagnetic Induction.
• Torque on a current loop.
• The magnetic dipole.
• Ampere's Law. Solenoids & Toroids Current loop as a magnetic dipole.
• AC – Generator.

6.2 Graphic Organizer
VII. General Objective(s)

To enable students to

- Understand the origin of currents, both direct and alternating current; the function and roles of the various devices and components such as resistors, capacitors, transformers etc. in electrical circuits;
- Understand, analyse and design various circuits diagrams;

VIII. Specific Learning Objectives

(Instructional Objectives)

<table>
<thead>
<tr>
<th>Content</th>
<th>Learning objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit 1: Electric Charges (20 hours)</strong></td>
<td>After Completing this section you would be able to:</td>
</tr>
<tr>
<td>• Conductors and Insulators;</td>
<td>• Differentiate between conductors and insulators;</td>
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<tr>
<td>• Coulomb’s Law</td>
<td>• Explain charging processes</td>
</tr>
<tr>
<td>• Electric Field</td>
<td>• State Coulomb’s law and solve problems based on it;</td>
</tr>
<tr>
<td>• Dipole moments</td>
<td>• Define an electric field and calculate dipole moments, potential energy and torque of an electric dipole;</td>
</tr>
<tr>
<td>• Flux and Electric Field</td>
<td>• Perform simple experiments of interaction between charged objects</td>
</tr>
<tr>
<td>• Gauss’ law and Coulomb’s law</td>
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</tr>
<tr>
<td>- Cylindrical symmetry</td>
<td>• State, derive and use Coulomb’s law to solve problems about electric field and electric potential</td>
</tr>
<tr>
<td>- Planar symmetry</td>
<td>• State and derive Gauss’ law</td>
</tr>
<tr>
<td>- Spherical symmetry</td>
<td>• Write the differential form of Gauss’ flux law</td>
</tr>
<tr>
<td>• A charged isolated conductor</td>
<td>• Use Gauss’ law to a number of kinds of charge distributions in space having high symmetry (spherical, cylindrical, and uniform-plane distribution)</td>
</tr>
<tr>
<td><strong>Unit 2: Flux and Electric Field (10 hours)</strong></td>
<td></td>
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</tbody>
</table>
Unit 3: Electric Potential (15 hours)

- Equipotential surfaces. V=V(E)
- V due to
  - A point charge
  - Electric dipole
  - Continuous charge distribution
- E=E(V) due to isolated conductor
- Van de Graaff accelerator

- define an electric potential and draw equipotential surfaces;
- derive expression for potential and calculate the potential of a point charge, and of a point charge distribution
- write relation between potential and electric field.
- explain the principles of a Van der Graaff generator and its applications

Unit 4: Capacitance (C) (15 hours)

- Calculating capacitance of
  - A parallel plate capacitor
  - A cylindrical capacitor
  - A spherical capacitor
- Capacitors in parallel and in series
- Storing Energy in an electric field
- Capacitors with dielectric field

- derive the expression for calculating capacitance
- explain how a capacitor stores energy in an electrical field
- explain the effect of a dielectric on capacitance
- derive expression for capacitance for combinations of capacitors, and use expressions for calculation
- derive different forms of expression for electrostatic energy stored in capacitors
- apply ideas about dielectrics to problems of simple parallel plate capacitor, filled between plates with dielectric materials; and to relate susceptibility to the dielectric constant

Unit 5: Direct Current (20 hours)

- Basic Concepts. The Schematic Diagram Kirchoff's Laws
- Resistivity. Equations with Multiple unknowns
- Mesh Analysis Equivalent Circuits Maximum Power Transfer
- Power Transfer Efficiency

- derive the equation for the current density
- explain the physical basis of Ohm’s law and use Ohm’s law in solving various problem of resistors connected in parallel and in series
- state and use the Kirchoff’s laws in circuit analysis
- perform mesh analysis of equivalent circuits
Unit 6: Magnetism (20 hours)

- Magnetic field, magnetic flux, flux and density.
- The magnetic force on a current-carrying wire.
- Moving charge in a magnetic field
- The Oscilloscope. Faradays’ law and electromagnetic Induction.
- Torque on a current loop.
- The magnetic dipole
- Ampere's Law. Solenoids & Toroids
  Current loop as a magnetic dipole
- AC – Generator

- define the terms: magnetic field, magnetic flux and flux density
- explain and draw magnetic field lines associated with current carrying conductors, and explain the principles of instruments based in it;
- explain the principles of an oscilloscope;
- state, explain and use Faraday’s law of electromagnetic induction;
- derive expression for force on a current-carrying wire in a magnetic field
- relate the force (F) to velocity (v), charge (q) and magnetic field (B)
- demonstrate magnetic field and interaction using magnets, and current-carrying wire, show the influence of the magnetic field by a moving charge using a oscilloscope, and demonstrate the electromagnetic induction/ Faraday’s law using simple materials
- derive expression for torque on current loop and apply the expression to calculate related problems
- define magnetic dipole
- write and apply the expression for dipole moment for calculation
- state, and use Ampere’s law
- derive and apply expressions for magnetic fields in solenoids & Toroids
- Explain the generation of alternative current/ voltage using a.c generator
IX. Pre-Assessment One

9.1 Rationale

To provide an opportunity for the learner to reflect on what were done while at school and therefore it will provide a starting point of the learning expected in this module for the student. It also provides some background readings on some of the basic concepts needed for learning the module.

A body is positively charged when it has
(A) excess electrons
(B) excess protons
(C) excess neutrons
(D) equal number of protons and electrons

It is difficult to charge an insulator by friction when the environment is humid because
(A) moisture is a bad conductor
(B) an insulator can only be charged by induction
(C) charges leak away during moist condition
(D) electrons are firmly held to the atoms

What happens when two magnets of similar poles are brought close to one another?
(A) they will attract one another
(B) they will remain in fixed positions
(C) they will repel one another
(D) they will lose their polarities

The unit of potential is
(A) joues
(B) volts
(C) ohms
(D) ohm-metre
A neutral point in a magnetic field is where
(A) the resultant magnetic flux is maximum
(B) the magnetic lines of force cross one another
(C) the net magnetic flux is zero
(D) a piece of iron experiences a force

The capacitance of a capacitor may be increased by
(A) decreasing the amount of charge stored
(B) increasing the surface area of the plate
(C) increasing the voltage across the plate
(D) filling the space between the plates with a vacuum.

The p.d across the plate of a parallel plate capacitor is 12.0V. If the capacitance of the capacitor is 470 \(\mu\)F, calculate the energy stored
(A) 3.84 x 10\(^{-2}\)J,
(B) 2.82 x 10\(^{-3}\)J
(C) 1.0368 x 10\(^{-2}\)J
(D) 3.819 x 10\(^{-3}\)J

The magnitude of induced e.m.f in a coil may be increased by
(A) decreasing the number of coils
(B) increasing the rate of change of magnetic flux.
(C) winding a coil round a piece of copper
(D) moving both coil and magnet in the same direction with the same speed

A conductor of length 60 cm is placed in a magnetic field of 0.2 T. Calculate the force that the conductor experiences if the current through it is 3.0A
(A) 36 N
(B) 0.36 J
(C) 1.0 N
(D) 9.0 J

Calculate the electric field at a distance of 3.0cm on a positive test charge due to a charge of 2.0 x 10\(^{-6}\) C.
Take \(\frac{1}{4\pi\varepsilon_0}\) 9.0 x 10\(^9\) newton-m\(^2\)/coulomb
(A) 2.0 x 10\(^7\) N C\(^{-1}\)
(B) 6.0 x 10\(^7\) N C\(^{-1}\)
(C) 5.4 x 10\(^7\) N C\(^{-1}\)
(D) 4.05 x 10\(^11\) N C\(^{-1}\)
Two point charges of $4.0 \times 10^{-6}$ C and $-3.0 \times 10^{-6}$ C are 2.0 cm apart. Calculate the force between them.

(A) $-2.7 \times 10^2$ N  
(B) $-5.4 \times 10^2$ N  
(C) $2.7 \times 10^{-3}$ N  
(D) $5.4 \times 10^{-1}$ N

A proton moves with a speed of $4.0 \times 10^6$ ms$^{-1}$ along the x-axis. It enters a region where there is a field of magnitude 5.0 T, directed at an angle of $60^\circ$ to the x-axis and lying in the xy plane. Calculate the initial magnetic force and acceleration of the proton.

(A) $2.77 \times 10^{-12}$ N  
(B) $3.2 \times 10^{-12}$ N  
(C) $1.6 \times 10^{-12}$ N  
(D) $6.4 \times 10^{-13}$ N

An electric heater is constructed by applying a potential difference of 110 V to a nichrome wire of total resistance 5 Ω. Find the current carried by the wire.

(A) 0.6 A  
(B) 13.8 A  
(C) 3.4 A  
(D) 1.52 A

A battery of e.m.f 18 V is connected across three resistors of 3Ω, 6Ω, and 9Ω. Calculate the power dissipated in the 6Ω resistor.

(A) 36 W  
(B) 108 W  
(C) 54 W  
(D) 72 W

An uncharged capacitor of capacitance 5 μF, and resistor of resistance $8 \times 10^5$ Ω are connected in series to a battery of e.m.f 12 V. Find the time constant of the circuit.

(A) 12 s  
(B) 6 s  
(C) 4 s  
(D) 2 s.
Which of the following statements is true?
(A) The magnetic force is proportional to the charge of a moving particle
(B) When a charged particle moves in a direction parallel to the magnetic field vector, the magnetic force on the charge is a maximum.
(C) The magnetic force on a positive charge is in the same direction as that of the force on a negative charge moving in the same direction.
(D) The magnetic lines of force originate from a south pole and ends on a north pole

Which of the following is NOT correct?
(A) The force between charges varies as the inverse of their distance.
(B) Charge is conserved
(C) Charge is quantized
(D) Conductors are materials in which electric charges move quite freely.

Identify a statement which is NOT correct?
(A) The electric lines of force begins on positive charges and terminate on negative charges
(B) The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of charge
(C) No two field lines can cross The force between two charged bodies is inversely proportional to their product.
9.2 Answer Key

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9.3 Pedagogical Comment For The Learners

The module is structured such that one activity follows the other. It is recommended that you stick to this order, that is, concept of electric charge; flux and electric field; electric potential; capacitance; current electricity; and magnetism.

The module provides you with a set of instructions, tasks including questions that will lead you through the module. A set of resources and references that you may use during the study are provided. You are advised to make your notes as you go through the tasks and instructions. For good and effective learning, you need to execute the instructions first before looking into the possible solutions provided. Your resources include the internet, recommended text, working with colleagues.

The learning activities are also structured such that the theoretical elements are given first. The student’s learning activities are given later. You are therefore advised that for each part you study the theoretical part and the student’s activity concurrently for maximum output.
X. Key Concepts (Glossary)

Coulomb’s Law of Force

States that the force between two point charges at rest is directly proportional to the product of the magnitude of the charges, i.e., \( F \propto \frac{q_1 q_2}{r^2} \), and is inversely proportional to the square of the distance between them i.e. \( \frac{1}{r^2} \). Thus, Coulomb’s law in vector form becomes:

\[
\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}
\]

Electric Field

When an electric charge is placed at some point in space, this establishes everywhere a state of electric stress, which is called electric field. The space where charge influence can be felt, is called site of electric field. The electric field strength at a point is operationally defined as the force \( \vec{E} \) acting on a unit test charge \( q_0 \) at that point:

\[
\vec{E} = \frac{\vec{F}}{q_0}
\]

Electric Potential

The electrostatic potential at a point is the work done against the forces of the electric field in bringing unit positive test charge from a point at zero potential to the point.

Electric Dipole moment

The product of the magnitude of the magnitude of either charge of a dipole and the distance separating the two point charges.

Equipotential Surfaces

Describes points in an electric field which are at the same electrostatic potential. All equipotential points in a field, when joined together, form equipotential line or surface.

Direct Current

A steady flow of electric charge carriers in one direction only.

Alternating Current

A current flowing in a circuit which reverses direction many times a second; it is caused by an alternating e.m.f. acting in a circuit and reversing many times a second
Ohm’s Law

States that the voltage across an arbitrary segment of an electric circuit equals the product of the resistance by the current.

Current Density

is the flow of current per unit area. Symbolized by $\vec{J}$, it has a magnitude of $i/A$ and is measured in amperes per square metre. Wires of different materials have different current densities for a given value of the electric field $E$; for many materials, the current density is directly proportional to the electric field.

Gauss’s Law

States that the electric flux across any closed surface is proportional to the net electric charge enclosed by the surface. The law implies that isolated electric charges exist and that like charges repel one another while unlike charges attract. Gauss’s law for magnetism states that the magnetic flux across any closed surface is zero; this law is consistent with the observation that isolated magnetic poles (monopoles) do not exist.

Capacitance

The mutual capacitance of two conductors is a quantity numerically equal to the charge $q$ which it is necessary to transfer from one conductor to the other in order to change the potential difference between them by one unit. i.e. $C = \frac{q}{V}$

Magnetic Field

A magnetic field is one of the the constituents of an electromagnetic field. It is produced by current-carrying conductors, by moving charged particles and bodies, by magnetized bodies or by variable electric field. Its distinguishing feature is that it acts only on moving charged particles and bodies.

Magnetic Flux

The flux ($\Phi$) of a magnetic field through a small plane surface is the product of the area of the surface and the component of the flux density ($B$) normal to the surface. If the plane is inclined at an angle ($\phi$) to the direction of the magnetic field, and has an area ($A$), then $\Phi = BA \sin \phi$
Magnetic Dipole Moment (\( \vec{\mu} \))

For a current carrying loop the magnetic dipole moment is the product of the current, the area and the number of turns of the loop. It is measured in ampere meter\(^2\), i.e. \( \vec{\mu} = NI\vec{A} \).

The direction of (\( \vec{\mu} \)) lies along the axis of the loop, as determined by the right hand rule.

Kirchhoff’s laws

Kirchhoff’s laws are two general laws for calculating the currents and resistances in networks at junctions. These laws are obtained from the laws of conservation of energy and the law of conservation of charge.

a. Kirchhoff’s first law: It applies to circuit nodes (or junction) and states that in any network, the algebraic sum of the currents at any junction in a circuit is zero.

b. Kirchhoff’s second law: it applies to close circuits (meshes) and states that in any closed circuit, algebraic sum of the products of the current and resistances of each part of the circuit is equal to the total emfs in the circuit.

Amperes Circuital Law

The generalized law, as corrected by Maxwell, takes the following integral form:

\[
\oint_C \vec{H} \cdot d\vec{I} = \iint_S \vec{J} \cdot d\vec{A} + \frac{d}{dt} \iint_S \vec{D} \cdot d\vec{A}
\]

where in linear media

\[
\vec{D} = \varepsilon \vec{E}
\]

is the displacement current density (in amperes per square meter).

This Ampère-Maxwell law can also be stated in differential form:

\[
\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}
\]

where the second term arises from the displacement current.

http://en.wikipedia.org/wiki/Ampere%27s_law
XI. Compulsory Readings

Reading #1 MIT Open courseware


Abstract: Topics covered in this reading material include: Electric and magnetic field and potential; introduction to special relativity; Maxwell's equations, in both differential and integral form; and properties of dielectrics and magnetic materials

Rationale: This is one of several second-term freshman physics courses offered at Massachusetts Institute of Technology (MIT). It is geared towards students who are looking for a thorough and challenging introduction to electricity and magnetism.

Reading #2 Physnet Project


Abstract: This online physics course for first-year undergraduates focuses on electric charge; electric field; electric potential; capacitance; RC circuits; magnetic field; Ampere's, Bio-Savart's, and Lenz's Laws; electromagnetic induction; and electromagnetic waves.

Rationale: Interactive tutorials, lab experiments, well-illustrated lecture notes and explanations, pedagogical suggestions, and problem hints are all included in this course. The course is based on the text Fundamentals of Physics (5th edition) by Halliday, Resnick, and Walker. (TG) Copyright 2005 Eisenhower National Clearinghouse

Reading #3 Electrodynamics

Complete reference: Physics - Electrodynamics: Electromagnetic Field Theory

Abstract: This is a free electronic book on Electrodynamics

Rationale: This reference is recommended for the last two activities of this module.
List of Relevant Resources

**Summary**: This resource is a video show on electric charges  
**Rationale**: Provides student with additional source of information  

**Summary**: An excellent website providing Lectures on all topics of electricity and magnetism in the module is provided  
**Rationale**: The site provides essentially all the basic lectures on electricity an magnetism.

**Abstract**: Helpful website for the students to use while reading on their own.  

**Summary**: Video clip showing lectures on a number of topics in electricity and magnetism  
**Rationale**: A good opportunity to listen to someone lecturing on the topics to learn

**Summary**: The illustration and presentation of electrostatic is well handled.  
**Rationale**: The source supplements what a student needs quite well.

**Summary**: Quite a good treatment of capacitance  
**Rationale**: Essential concepts on capacitors have been ably demonstrated to help in the understanding of the concept

**Summary**: Good illustration of behaviour of charge  
**Rationale**: Provides good examples of treatment of Gauss’ law

**Summary**: Valuable demonstrations of forces between charges provided  
**Rationale**: The website provides a rich resource for learning electricity and magnetism
<table>
<thead>
<tr>
<th>Reference</th>
<th><a href="http://www.pha.jhu.edu/dept/lecdemo/videodiscs.html">http://www.pha.jhu.edu/dept/lecdemo/videodiscs.html</a></th>
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</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Video clip showing lectures on a number of topics in electricity and magnetism</td>
</tr>
<tr>
<td>Rationale</td>
<td>Useful for filling up the role of a physical lecturer while learning</td>
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<tbody>
<tr>
<td>Summary</td>
<td>Provides an example of a gold leaf electroscope for students to see and use</td>
</tr>
<tr>
<td>Rationale</td>
<td>A useful resource for students to use for learning electrostatic</td>
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XII. Useful Links

List of Relevant Useful Links

Title: Electric Charges
Abstract: good article on electric charges is available

Title: Electrostatics
Abstract: More relevant information on electrostatics is provided.

Title: Electric Field
URL: http://wikipedia.org/wiki/electric_field, 20/10/2006
Abstract: A good additional link to getting more information on electric field.

Title: Lectures on Electricity and manetism
URL: http://web.mit.edu/smcs/8.02/
Abstract: An excellent website providing Lectures on all topics of electricity is provided.

Title: Lectures on electricity and magnetism
URL: http://qemp.deas.harvard.edu:8182/students/lectures/specificlecture/?lectureID=4764#video, 24/12/2006
Abstract: Helpful website for the students to use while reading on their own.

Title: Gauss’ law
Abstract: Simplified discussion and presentation of Gauss law treated

Title: Electricity and magnetism
Abstract: Good resources for electricity and magnetism

Title: Electricity and magnetism
Abstract: Simplified discussion and presentation of Gauss law treated

Title: Electricity and magnetism
Abstract: The different aspects especially magnetic field have been well treated
XIII. Teaching And Learning Activities

Title: Concept of Electric Charge

You will require 30 hours to complete this activity. Only basic guidelines are provided for you in order to help you do the rest of the curriculum in the activity. Personal reading and work is strongly advised.

Specific Teaching and Learning Objectives

• Differentiate between conductors and insulators;
• Explain charging processes
• State Coulomb’s law and solve problems based on it;
• Define an electric field and calculate dipole moments, potential energy and torque of an electric dipole;
• Perform simple experiments of interaction between charged objects

Summary of The Learning Activity

A clear differentiation between conductors and insulators in terms of how they acquire electric charge will be accomplished by end of the study. This will lead you to stating and derivation of Coulomb’s law, stating the relation between both types of charges. These relations will be used for calculations. Expressions for volume density, and area density will also be derived. Coulomb’s law, and Gauss’ laws will be derived and applied in different situations.

Key Concepts

Electric charge
An electric charge is an attribute of matter that produces a force, just as mass causes the gravitational force, but unlike mass, electric charge can be either positive or negative.

Electric field \( \vec{E} \)
The electric field, \( \vec{E} \), is a vector quantity which gives, at every point in space, the force that would act on a unit positive charge that is placed at that point. Thus \( \vec{E} \) is related to the force, \( \vec{F} \), which acts on any charge \( q \) at any point by the equation.

\[
\vec{E} = \frac{\vec{F}}{q_o}
\]

This is the basic definition of electric field. The unit of \( \vec{E} \) is newtons per coulomb which is denoted as \((\text{NC}^{-1})\). The magnitude of \( \vec{E} \) is called the intensity of the electric field.
Electric field lines
The electric field lines describe the (vector) electric field in any region of space according to the following rules:

- The direction of electric lines drawn in space is the same as the direction of the field at each point.
- The density of lines in a given region is proportional to the magnitude of the field in that region. The density lines means the number of lines per unit area cutting a surface perpendicular to the direction of the lines at any given point.

It is a direct consequence of the Coulomb inverse-square law that all possible static field configurations can be described by lines in the fashion above, where all lines originate on positive electric charge and end on negative charges. Lines are thus continuous except at their sources and sinks on positive and negative charge respectively

- The number of lines originating or ending on charges is proportional to the magnitude of each charge.

An electric dipole
Is a pair of equal and opposite charges, $q$ and $-q$, separated by a distance $2a$.

Torque on dipole in external field
If the external field, $E$ is uniform and the dipole makes an angle $\theta$ with the field, the net torque about the centre of the dipole is: $\tau = 2aqE \sin \theta = pE \sin \theta$

$p$ is electric dipole moment

Key Words
- Charge
- Force
- Electric field
- Dipole
- Dipole moments
- Electric dipole
- Flux
- Inverse square law
Introduction to the Activity

The knowledge of the existence of electrostatic charge goes back at least as far as the time of ancient Greeks, around 600 B.C. We can repeat the observation of the Greeks by rubbing a rod of amber or hard rubber with a piece of fur. After this it will be found that small bits of paper or other light materials are attracted to the rod. No particular advance was made in the understanding of this phenomenon until about 1600, when William Gilbert, did a detailed study of the kinds of materials that would behave like amber. Other studies did reveal that matter is made up of exactly equal mixtures of both negative and positive charges. The implication of this is that there is usually no net electric force of consequence between separate bodies. The electric force is responsible for holding individual atoms together, and holding the groups of atoms together to form solid matter. We are usually unaware of the presence of charge because most bodies are electrically neutral, that is, they contain equal amounts of positive and negative charge.

For example, a hydrogen atom consists of a single proton with a single electron moving around it. The hydrogen atom is stable because the proton and electron attract one another. In contrast, two electrons repel one another, and tend to fly apart, and similarly the force between two protons is repulsive. The magnitude and direction of the force between two stationary particles each carrying electric charge, is given by Coulomb’s law.

Using Coulomb’s law the electric field can be defined, and thereafter we are able to solve problems on electric dipole moments, potential energy, and torque of an electric dipole.
Detailed Description of The Activity
(Main Theoretical Elements)

Tasks 1 : Electric Charge

Task 1.1 : Conductors and insulators

Materials are divided into three categories:

- conductors - metals, for example
- semi-conductors - silicon is a good example
- insulators - rubber, wood, plastic for example

The notion that Charge is quantized means that charge comes in multiples of an indivisible unit of charge, represented by the letter e. In other words, charge comes in multiples of the charge on the electron or the proton. Both proton and electron have the same size charge, but the sign is different. A proton has a charge of +e, while an electron has a charge of -e.

To express the statement "charge is quantized" in terms of an equation, we write:

\[ q = ne \]

\( q \) is the symbol used to represent charge, while \( n \) is a positive or negative integer, and \( e \) is the electronic charge, of magnitude \( 1.6 \times 10^{-19} \) Coulombs (C). The unit of charge is coulomb, and its symbol is C

Task 1.2 : Coulomb’s law

This gives a relation between two charges \( Q_1 \) and \( Q_2 \) which are at a separation \( r \) apart. Experiments show that the forces between two bodies obey an inverse square law and that the force is proportional to the product of the charges. Simply, Coulomb’s law states

The force between two charges at a distance, \( r \) apart is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them

Mathematically this is written as

\[ F = \frac{q_1q_2}{4\pi\varepsilon_0r^2} = K\frac{q_1q_2}{r^2} \]

(1)

where, \( K = \frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \text{Nm}^2\text{C}^{-2} \) = a constant and \( \varepsilon_0 \) is permittivity of free space.

The following are useful relations for Charge distributions. Study them and use them for calculations. These relations are found in most standard text books.
For charge per unit volume, the volume density is: \( \rho = \frac{dq}{dV} \text{ C/m}^3 \)

For charge per unit area, the area density is: \( \sigma = \frac{dq}{dA} \text{ C/m}^2 \)

For charge per unit length, the linear density is: \( \mu = \frac{dq}{dl} \text{ C/m} \)

In the special cases where density is uniform over a region \( \rho \frac{Q}{V} \text{ C/m}^3 \)

**Task 1.3: Electric Field**

(a) We are able to write expression for electric field by using what we have learnt about Coulomb’s law. From definition of electric field, \( E \) we have

\[
\text{Electric field (E)} = \frac{\text{Electric force (F)}}{\text{Test charge}} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r^2} \quad q \text{ is the test charge.}
\]

(b) Using the principle of superposition, the value of \( \vec{E} \) due to \( n \) discrete charges \( q_1, q_2, q_3, \ldots q_n \) at rest is

\[
\vec{E} = \sum_{i=1}^{n} \frac{q_i q_j}{r_i^2 \hat{r}}
\]

(c) For a body of continuous charge, the electric field at a distance \( r \) away is

\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2 \hat{r}}
\]

**Unit**
The unit of charge is the Coulomb. It is denoted by a letter \( C \)

**Unit analysis**

\[
F = \frac{Kq_1 q_2}{r^2}
\]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>SI unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>( F )</td>
<td>N (Newtons)</td>
</tr>
<tr>
<td>Charge</td>
<td>( Q )</td>
<td>C (Coulombs)</td>
</tr>
<tr>
<td>Displacement/distance</td>
<td>( r )</td>
<td>m (metres)</td>
</tr>
<tr>
<td>Constant</td>
<td>( K )</td>
<td>N·m²·C⁻²</td>
</tr>
</tbody>
</table>
Task 1.4 Electric Dipole Field

The electric field of an electric dipole can be constructed as a vector sum of the point charge fields of two charges as shown below. The direction of the electric dipole is as shown.

![Figure 1.1 Direction of electric dipole](image)

For a dipole, the dipole field at points in equatorial plane, a distance $r$, from the centre is given by:

$$ E = \frac{p}{4\pi \varepsilon_0 (a^2 + r^2)} \text{ Newton/Coulomb} $$

Task 1.5: Gauss’ Theorem

Gauss’ flux theorem actually embodies nothing more than the validity of the electric field-lines point of view, and is thus a direct consequence of Coulomb’s inverse-square law. The theorem is stated mathematically as follows:

$$ \oint \int_{\text{Closed Surface}} E \cos \theta \, ds = \sum_{i} \frac{q_i}{\varepsilon_0} $$  \hspace{1cm} (4)

That is, the surface integral of the normal component of $E$ over a closed surface equals the sum of charges inside the enclosed volume divided by $\varepsilon_0$.

By Gauss’ law, it can be shown that the field $E$ due to an infinite plane of sheet is also given by

$$ E = \frac{\sigma}{2\varepsilon_0} $$  \hspace{1cm} (5)
Students Activity

Task 1.1.1 Conductors and Insulators

Refer to the books listed in the reference section and other references as well as the Links provided to make notes about conductors and Insulators and do the following:

- Collect insulating materials such as glass, silk, fur and ebonite.
- Rub glass (Bic pen) and silk or glass on your air.
- Bring the glass near a piece of paper. You will notice that: The paper will be attracted to the glass.

Alternatively

- Switch on a TV
- Bring a piece of paper near the screen. You will also notice that: the piece of paper will be attracted to the screen.

These two observations serve to demonstrate the presence of electric charge.

When you bring similar charges together you will observe that similar charges attract, while dissimilar charges repel.

TRY THIS OUT!

Use relevant literatures and note down the explanation of how a body becomes charged. The theory is that a body is charged when it has excess protons or excess electrons.

It may not be easy in our local setting to have access to an equipment which can enable you to demonstrate Coulomb’s law. However, you can do this:

- Charge a gold leaf electrocope positively by induction.
- Similarly, charge another conducting sphere which is fixed on an insulating stand.
- Bring the charged sphere near the cap of a gold leaf electroscope. The leaf will diverge.
- Again if you bring another body which is negatively charged, the leaf will collapse.
- Explain what you observe. These two observations serve to show that like charges repel and unlike charges attract each other.
- What is the SI unit of charge? Use methods of dimension to determine this.
Task 1.2.1  Coulomb’s law

Refer to Arthur F. Kip (1969). Pp. 3-21 or any relevant book on electricity and magnetism
Make short notes on Coulomb’s law
From the mathematical form of Coulomb’s law:
derive the unit of $\varepsilon_0$ in SI $F = \frac{1}{4\pi\varepsilon_0 r^2} Q_1 Q_2$

Follow the example given below for the use of Coulomb’s law

Numerical Example

Four charges $q_1$, $q_2$, $q_3$ and $q_4$ of magnitudes $-2.0 \times 10^{-6}$ C, $+2.0 \times 10^{-6}$ C, $-2.0 \times 10^{-6}$ C, and $+2.0 \times 10^{-6}$ C are arranged at the corners of a square ABCD respectively. The sides of the square are of length 4.0 cm. What is the net force exerted on charge at B by the other three charges?

Solution

The solution to this problem needs a clear diagram to be drawn as shown in Fig. 1.2. The forces on charge $q_2$ at B are as shown.

![Diagram](image)

Figure 1.2

To solve for the net force on charge $q_2$ at B, we have to determine first the forces $F_{BA}$, $F_{BC}$, and $F_{BD}$ between charges $q_1$ and $q_2$; $q_1$ and $q_3$; $q_1$ and $q_4$ respectively.
Remember that force is a vector, and any time you have a minus sign associated with a vector all it does is tell you about the direction of the vector. If you have the arrows giving you the direction on your diagram, you can just drop any signs that come out of the equation for Coulomb's law.

Using Coulomb’s law equation,

\[ F = \frac{kq_1 q_2}{r^2} \]

\[ k = 8.99 \times 10^9 \text{ Nm}^2\text{C}^{-2} \]

\[ r = 4.0 \times 10^{-2} \text{ m} \]

\[ F_{BA} = \frac{8.99 \times 10^9 \times (-2.0 \times 10^{-6}) \times (2.0 \times 10^{-6})}{(4.0 \times 10^{-2})^2} = -22.475 \text{ N} \]

\[ +22.475 \text{ in the direction shown on the diagram} \]

\[ F_{BC} = \frac{8.99 \times 10^9 \times (-2.0 \times 10^{-6}) \times (2.0 \times 10^{-6})}{(4.0 \times 10^{-2})^2} = -22.475 \text{ N} \]

\[ +22.475 \text{ in the direction shown on the diagram} \]

\[ F_{BD} = \frac{8.99 \times 10^9 \times (+2.0 \times 10^{-6}) \times (2.0 \times 10^{-6})}{(5.66 \times 10^{-2})^2} = +11.2375 \text{ N} \]

\[ +11.2375 \text{ in the direction shown on the diagram} \]

The net force on q2 is obtained by adding \( F_{BA} \), \( F_{BC} \), and \( F_{BD} \) vectorally. By Pythagoras theorem, the combined effect, \( F_f \), of \( F_{BA} \) and \( F_{BC} \) is given by

\[ (F_f)^2 = (F_{BA})^2 + (F_{BC})^2 = 22.4752 + 22.4752 \]

\[ F_f = 31.78 \text{ N} \]

Directed along the diagonal from B towards D

Note that \( F_f \) and \( F_{BD} \) act along the same line, but in opposite directions. The net force \( F_{net} \) on q2 is therefore given by

\[ F_{net} = F_f - F_{BD} = 31.78 - 11.2375 = 20.55 \text{ N} \]

Directed along the diagonal from B towards D

Use the example above and do the following

Two charges of \( 2.0 \times 10^{-6} \text{ C} \) and \( 4.0 \times 10^{-6} \text{ C} \) are placed 3.0 cm apart in a vacuum. Find the force acting between them. (Ans. 8.0 N)
**Task 1.3.1 Electric field**

Read about electric field and then make short notes.
Check that the dimensions in the equation in the equation below is correct

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2} \]

Use this expression and find the field due to a charge \(4.0 \times 10^{-6} \text{ C} \) at a distance of 3.0 cm away. Take \( \varepsilon_0 \) is permitivity of a free space.

\[ k = \frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \quad \text{N-m}^2\text{C}^{-2} \]

An example, of electric field due to a positive and negative charge is shown in Fig. 1.3

![Figure 1.3 Field lines due to a positive and negative charge](image)

The field lines for one positive point charge and one negative point charge. The field lines flow out of the positive charge and into the negative charge.

Sketch the following electric field lines due to

i. a point charge
ii. electric dipole
iii. two similar charges,
iv. charged plate (disc)
Example: Field out from a long uniform charged rod

Here we give an example on how to show how electric field out from a long uniform charged rod can be calculated.

Let the linear charge distribution be $\mu$, the electric field at point P be $E$, at a distance $a$ along the perpendicular bisector of the rod.

When total charge on rod is $Q$, then $\mu = \frac{Q}{L}$.

(a) show this

The component of the field at P due to the element of charge $\mu \, dx$ is

$$dE = \frac{\mu \, dx}{4 \pi \varepsilon_0 \, r^2} \text{ by Coulomb's law.}$$

(b) Why is Coulomb's law being stated here?

Since $\frac{x}{a} = \tan \theta$ and $\frac{a}{r} = \cos \theta$, we have $dx = a \sec^2 \theta \, d\theta$ and $r = \frac{a}{\cos \theta}$

Thus, $dE = \frac{\mu}{a} \frac{1}{4 \pi \varepsilon_0} \, d\theta$

Note that the x components, $dE$ at P, adds up to zero.

(c) Explain this statement

We therefore take the sum of the y components of each $dE$ to obtain the required vector sum. Let such a component be $dE'$

Thus, $dE' = \frac{1}{4 \pi \varepsilon_0} \frac{\mu}{a} \cos \theta \, d\theta$

The total field $E'$ at P for a very long rod is then obtained from

$$E' = \frac{2}{4 \pi \varepsilon_0} \frac{\mu}{a} \int_0^{\pi/2} \cos \theta \, d\theta = \frac{2\mu}{4 \pi \varepsilon_0 \, a} \text{ newtons/coulomb (Check this!)}$$

This shows us that the field falls off in the ratio $1/a$ as we move away from the rod.
Task 1.4.1  Dipole moment

Carry out the following exercises after reading about a dipole

Write an expression for a dipole moment.

Calculate the dipole moment for two charges of $3.0 \times 10^{-6}$ C and $-3.0 \times 10^{-6}$ C if the separation between them is 2.0 cm. (Ans. $6.0 \times 10^{-8}$ C-m)

Explain why the net force on a dipole in a uniform field $E$ is zero.

(d) Recall the definition of torque and show that

The magnitude of the torque in an electric field with respect to the centre of the dipole is the sum of the two forces times their lever arms.

$$|\tau| = 2q |E| a \sin \theta = |p| |E| \sin \theta$$

(e) Using this finding, explain why the torque is given by

$$\tau = p \times E$$  Newton – metre.

(SHOW THE STEPS FOLLOWED IN DERIVING THIS)

Example:  The following example gives you calculation on electric dipoles.

Follow each step carefully.

---

**Figure 1.5**

Let point $P$ be in the equatorial plane of the dipole, at a distance $r$, from the centre of the dipole. Dipole electric field at points in the equatorial plane is given by

$$E = \frac{q}{4\pi \varepsilon_0} \frac{1}{a^2 + r^2} \cos \theta$$

(a) Show that the distance from either $+q$ or $-q$ to $P$ is $(a^2 + r^2)^{1/2}$?

$$= \frac{2q}{2\pi \varepsilon_0} \frac{1}{(a^2 + r^2)^{1/2}}$$
Dipole field at points in equatorial plane, a distance \( r \), from the centre

\[
\frac{p}{4\pi\varepsilon_0 \left( a^2 + r^2 \right)^{3/2}} \quad \text{Newton/coulomb}
\]

Remember that \( p = 2q \)

**Task 1.5.1  Flux and electric field**

Read widely on Gauss’s law and do the following tasks. (Use the following references: Grant I S; W. R, Philips ,1990.; Serway, (1986) ; Dick, G et al, 2000) or any other relevant text and links provided.

The mathematical form of Gauss’ law is

\[
\oint_{\text{Closed Surface}} E \cos \theta \, ds = \sum_i \frac{q_i}{\varepsilon_0}
\]

In words, Gauss’ law states that “the electric flux across any closed surface is proportional to the net electric charge enclosed by the surface”. The law implies that isolated electric charges exist and that like charges repel one another while unlike charges repel. While Gauss’ law for magnetism states that the magnetic flux across any closed surface is zero. This law is consistent with the observation that isolated magnetic poles (monopoles) do not exist.

How does the field \( E \) come in the above expression? (To answer this make a brief note on how this expression is derived)

An example: Use of Gauss’ law for a single point charge

Gauss’ law applies to any charge contribution but let us apply it now to the simplest case of a single point charge

We start by constructing a spherical Gaussian surface of radius \( r \) around a charge \( +q \). This is followed by take a small area \( dA \) on the Gaussian surface. The vector area \( dA \) points radially outward, as does the electric field \( E \) at this point.

The electric flux through this small area is

\[
d\phi = E \cdot dA = E dA \cos 0^\circ = E dA
\]

From the spherical symmetry, all such small area elements contribute equally to the total.

\[
\theta = \oint E dA = \oint E dA = E(4\pi r^2)
\]
Explain how the term \((4\pi r^2)\) comes in. According to Gauss law

\[
\phi = E(4\pi r^2) = \frac{q_{\text{encl}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}, \text{ because } q_{\text{encl}} = q
\]

Solving for the electric field gives

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2}.
\]

This expression is just Coulomb’s law.

Example: Use of Gauss’ law applied to an infinite plane of charge.

Figure 1.7  Electric Field due to infinite plane of charge

Here we want to show that for an infinite sheet which carries a uniform charge density \(s\), the field

\[
E = \frac{\sigma}{2\varepsilon_0}
\]

Procedure

By symmetry the resultant E-field must have a direction normal to the plane and must have the same size at all points of the same distance from the plane.

Take as a Gaussian surface a cylinder of cross-sectional area \(A\) and height \(2h\).

Flux is only non-zero through the ends of cylinder.

Read and make notes about this topic and explain why the flux is only non-zero at the ends of the cylinder (Use the following references: Grant I S; W. R. Philips, 1990; Serway, (1986); Dick, G et al., 2000) and any other relevant text and links.

Read the following statements.

If field at cylinder ends is \(E\) then total flux \(\Phi = 2EA\).

Charge enclosed is area \(A\) x charge density \(\sigma = A\sigma\)

Hence from Gauss's law

\[
2EA = \frac{A\sigma}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}
\]

Explain how the expression of the last three lines are obtained.
Two examples have now been given to you. Use Gauss’ law and use similar arguments to derive and show that the field due to spherical but non-point charge is given by

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

Explain all the steps.

**Experimental work**

You can work in a group with colleagues.

Problem: How can a body acquire charge?

Hypothesis: Two insulators attract/repel one another when not rubbed together.

Equipment:
- Fur
- Pieces of paper
- Glass rod
- Ebonite
- Polythene

**Procedure**

(a) Charging by friction

Requirements: glass rod, silk cloth, and a piece of paper.

Step I

Rub the glass rod with silk cloth. While keeping them together, bring both of them close to a piece of paper. What do you observe?

Step II

Separate the glass rod and silk. Bring only one of them, say glass rod, close to a piece of paper. What do you observe?

Why is it that in step I nothing happens to the piece of paper but in step II, the paper is attracted to the glass?

**Response**

In step I, when the glass rod and silk are together, they are essentially a neutral body. In step II, the paper is attracted to the glass rod because the rod has a net positive charge which induces a negative charge on the paper. The consequence of this, leads to an attractive force which causes the paper to move to the glass.

**Charging by induction**

Requirements:

Use a negatively charged object and an initially-uncharged conductor (for example, a metal ball on a plastic handle).

Bring the negatively-charged object close to, but not touching, the conductor. Explain what happens at this stage.

Connect the conductor to ground. What is the importance of connecting the conductor to the earth?

Remove the ground connection. This leaves the conductor with a deficit of electrons.
Remove the charged object. The conductor is now positively charge. Explain how this statement may be verified.

The gold electroscope is shown below

[Image]


Further Task
Use the gold leaf electroscope shown and explain how one may use it as a teaching tool for electrostatics at school.

**Learning activities**

You are provided with an example in how you may use Coulomb’s law is solving numerical problems. What are the important aspect of Coulomb’s law?

**Formative Evaluation 1**

1. Use the concept of corona discharge to explain how a lightening conductor works.
2. Derive an expression for the field of an electric dipole along axis, and normal to axis.
3. Use the principles applied in deriving electric field out from a long uniform charged rod to derive electric field for an electric dipole in any direction; and electric field due to plane distribution of charges

[Diagram]

Figure 1.8
4 Find the electric field at a distance z above the midpoint of a straight line segment of length 2L which carries a uniform line charge \( \lambda \).

5 Use Gauss’ theorem for the following situations of high symmetry and derive:
   
   (i) Field of a uniform spherical shell of charge
   
   (ii) Field of a spherical charge distribution
   
   (iii) Field in region of charged cylindrical conductor
Activity 2

Title: Electric Potential

You will require 15 hours to complete this activity. Only basic guidelines are provided for you in order to help you do the rest of the curriculum. Personal reading and work is strongly advised here.

Specific Teaching and Learning Objectives

• Should define an electric potential and draw equipotential surface;
• Derive expression for potential and calculate the potential of a point charge, and of a point charge distribution
• Explain the principles of Van der Graaff generator and its applications.

Section Expectation

You will define related terms and relate potential to electric field and discuss: equipotential surfaces, potential due to a point charge, electric dipole, continuous distribution; electric field due to isolated conductor and van de Graaff generator.

Summary of The Learning activity

The definition and derivation of electric field potential and potential will be learnt and then used to solve related problems. Besides explanation and discussion of the principles of Van der graaff generator will be made.

Key concepts

Equipotential surface-is a surface on which the potential, or voltage, is constant. Electric field lines are always perpendicular to these surfaces, and the electric field points from surfaces of high potential to surfaces of low potential. Suppose, for example, that a set of surfaces has been chosen so that their voltages are 5 V, 4 V, 3 V, 2 V, etc.. Then since the voltage difference between neighboring sheets is constant ( \( \Delta V = 1V \) ) we can estimate the magnitude of the electric field between surfaces by the formula

\[
\text{Electric Potential Energy (PE)} = qEd
\]

where \( q \) is the charge on an object

\( E \) is electric field produced by \( Q \), and

\( d \) is the distance between the two charges

Voltage- is also related to force.

\[
V = Ed = \frac{Fd}{q} = Wq
\]
( W = Fd - force times displacement in the direction of force is work (J))

A high voltage -means that each individual charge is experiencing a large force. A low voltage -means that each individual charge is experiencing a small force.

Van der Graaff Generator- Is a high-voltage electrostatic generator that can produce potential of millions of volts

**Key Terms**

Work  
Electric potential  
Voltage  
Electric Dipole Potential

**Introduction to the Activity**

The principle point of this activity is to extend the concept of electric field to potential. It has already been shown that a static electric field is conservative, which means that the work to move a charge from one position to another against the force of field is independent of the path taken. A further consequence is that the circulation , that is, the line integral of the field taken around any closed path, is always zero.thus we write

$$\oint E \cdot dl = 0$$  \hspace{1cm} (2.1)

This property of zero circulation provides a useful method of characterizing the conservative nature of static field, and is a powerful conceptual tool for solving certain kinds of problem.
Detailed Description of The Activity
(Main Theoretical Elements)

Task 2.1 Electric Potential

(a) From your earlier work you know that:
- work must be done on or to the charge in order to bring them close to each other
- since work has to be done on or by a charge, it has potential energy.

(b) From mechanics we know that, gravitational force between two masses, m₁, m₂ at a separation R apart is:

\[ F = G \frac{m_1 m_2}{R^2} \]  \hspace{1cm} (2.2)

and, gravitational potential energy PE is given by:

\[ PE = mgh; \]  \hspace{1cm} (2.3)

where m is mass, g is acceleration due to gravity, and h is the height.

(c) Similarly electrical potential V is given by

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r} \]  \hspace{1cm} (2.4)

where q₁ and q₂ are the charges at a separation r.

(d) Potential and electric are related as follows:

\[ E_x = -\frac{dv}{dx} \]  \hspace{1cm} (2.5)

Potential is measured in Joules (J).

Task 2.2 Electric potential due to point charge

The potential due to a charge Q at a distance r away is given by

\[ V = \frac{Q}{4\pi\varepsilon_0 r} \]  \hspace{1cm} (2.6)

Task 2.3: Potential due to several point charges

Figure 2.1
The contribution of potential, \( V_0 \) at the origin, \( O \) of each charge \( q_1 \) and \( q_2 \) is

\[
V_0 = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_1 + q_2}{r_1 + r_2} \right)
\]  

(2.7)

Thus the general expression for the potential at a given point in space due to a distribution of point charges is

\[
V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_i}
\]

(2.8)

Potential is a scalar quantity, and its unit is volts (V)

If the distribution is continuous, the expression for potential in terms of volume density of charge \( \rho \) which may vary from point to point is

\[
V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho dv}{r}
\]

(2.9)

**Task 2.4 : Electric Dipole Potential**

Potential due to a dipole at point P shown in Fig.2.2 is given by equation (2.10) similarly the electric field at P is given by equation (2.11).

\[
V = \frac{1}{4\pi\varepsilon_0} \frac{p \cos \theta}{r^2} \quad \text{and}
\]

(2.10)

Electric field is given by

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3} \left( \cos^2 \theta + 1 \right)
\]

(2.11)

**Task 2.5 Van der Graaff Generator**

Is a high-voltage electrostatic generator that can produce potential of millions of volts
Students activity

Task 2.1.1

(a) Refer to Arthur F Kip (1969); Serway (1986) and Grant (1990) and make notes on potential plus the links provided

(b) Show that electric potential energy  
   HINT: Remember that work = force times distance  
   \[ V = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r} \]

Task 2.2.1

(a) Given that potential of a charged metal sphere is essentially the same as that due a point cahrge Q at a point P a distance r away, show the steps required that potential of a charged metal sphere is  
   \[ V = \frac{Q}{4\pi\varepsilon_0 r_0} \]  
   where \( r_0 \) is the radius of the sphere

Task 2.3.1

Refer to Arthur F Kip (1969); Serway (1986) and Grant (1990) and make notes on potential
Identify the symbols used in equation (2.9)
Show that the expression \( \rho \, dV \) gives the quantity of charge
What is the difference between electric potential and electric potential energy?

Task 2.3.2

Use equations (2.5) and (2.9) and show that electric field at P along the axis of a uniformly charged ring in Fig. 2.3 is given by

\[ dq \]

\[ \frac{-\lambda}{2\pi}\ln\frac{x^2+a^2}{r^2} \]

Figure 2.3
Task 2.4.1 Example

Using Fig 2.2, The potential of an electric dipole can be found by superposing the point charge potentials of the two charges as follows

By definition

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \]

Thus potential at P due to the dipole is given by

\[ V = \frac{1}{4\pi \varepsilon_0} q \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

For cases where \( r \gg d \), this can be approximated by

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{p \cos \theta}{r^2} \]

Where \( p = qd \) is the dipole moment. The approximation made in the last expression is that

When \( r \gg d \) then

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{p \cos \theta}{r^2} \]

And

Electric field, \( E \) is given by

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{p}{r^3} \left( 3 \cos^2 \theta + 1 \right) \]

Identify and write down the assumptions made in writing equations 2.10
Show how equation 2.11 is derived.

Task 2.5.1 Van der Graaff Generator

You could work in a group for this activity

Purpose

To have a clear identification and understanding of the different parts of a van der Graaff generator

Apparatus

A complete van der graaff generator

(a) Dismantle a van der Graaff generator and see whether you can identify the different parts
(b) When you are satisfied with the identification then reassemble the generator.
(c) Use now the diagram of a Van der Graaff generator shown in Fig. 2.3 and
   (i) label the different parts marked with numbers
   (ii) describe the functions of each part and hence
   (iii) explain how the generator works.
(d) describe practical applications of Van der Graaff generator


Figure 2.3

You are provided with a numerical example to show you how the expressions for potential can be used for calculations.

**Task 2.1 Calculation of Electric potential for a system of Charges**

Three point charges, Q1, Q2, and Q3, are at the vertices of a right-angled triangle, as shown above. What is the absolute electric potential of the third charge if Q1 = -4.0 \times 10^{-6} \text{ C}, Q2 = 3.0 \times 10^{-6} \text{ C}, Q3 = 2.0 \times 10^{-6} \text{ C}. If Q3, which is initially at rest, is repelled to infinity by the combined electric field of Q1 and Q2, which are held fixed. Find the final kinetic energy of Q3.
Solution: The absolute electric potential of $Q_1$ due to the presence of $Q_1$ is

$$V_1 = KE \left( \frac{q_1}{5} \right) = 9 \times 10^3 \times \frac{-4 \times 10^{-6}}{5} = -7.2 \times 10^3 \text{V}$$

Similarly, the absolute electric potential of $Q_1$ due to the presence of $Q_2$ is

$$V_2 = KE \left( \frac{q_2}{3} \right) = (9 \times 10^3) \times \frac{3 \times 10^{-6}}{3} = 9 \times 10^3 \text{V}$$

The net absolute potential of $Q_3$ is simply the algebraic sum of the potentials due to the $Q_1$ and $Q_2$ taken in isolation. Thus,

$$V_3 = V_1 + V_2 = 1.8 \times 10^3 \text{V}$$

The change in electric potential energy of $Q_3$ as it moves from its initial position to infinity is the product of $Q_3$ and the difference in electric potential $-V_3$ between infinity and the initial position. Therefore

$$\Delta V = -Q_3V_3 = -2.0 \times 10^{-6} \times 1.8 \times 10^3 = 3.6 \times 10^{-3} \text{J}$$

This decrease in the potential energy $Q_3$ equals to increase in its kinetic energy, since the initial kinetic energy of $Q_3$ is zero.

Thus, kinetic energy $= 3.6 \times 10^{-3} \text{J}$

Formative Evaluation 2

A dipole of charge $\pm q$ and separation $\ell$ (dipole moment $p = q\ell$) is placed along the x-axis as shown below

(i) Using the expression for the potential $V$ at a point charge, calculate the work necessary to bring a charge $+Q$ from far away to a point $S$ on the $x$-axis, a distance $a$ from the the centre of the dipole.

(ii) What is the potential $V_s$ of point $S$ (in the absence of the charge $Q$)?

(iii) Write a simple approximate expression for $V_s$, good for $a >> \ell$.

(iv) Use the expression for $V_s$ to find the magnitude and direction of the electric field at the point $S$. Find the orientation of the equipotential surface at the point $S$. (You may use Kip F, 1986) for more information)
Activity 3

Title: Capacitance

You will require 15 hours to complete this activity. Only basic guidelines are provided for you in order to help you do the rest of the curriculum in the activity. Personal reading and work is strongly advised here.

Specific Teaching and Learning Objectives

- Derive the expression for calculating capacitance
- Explain how a capacitor stores energy in an electrical field
- Explain the effect of a dielectric on capacitance
- Derive expression for capacitance for combinations of capacitors, and use expressions for calculation
- Derive different forms of expression for electrostatic energy stored in capacitors
- Apply ideas about dielectrics to problems of simple parallel plate capacitor, filled between plates with dielectric materials; and to relate susceptibility to the dielectric constant

Summary of the Learning Activity

Derivations for expressions for combinations of capacitors; relation between capacitance, voltage and charge; and electrostatic energy stored in capacitors will be made. These expressions will be used for solving numerical problems.

Key Concepts

Capacitors - are short term charge-stores, a bit like an electrical spring. They are used widely in electronic circuits. It consists of two metal plates separated by a layer of insulating material called a dielectric.

Two types of capacitors: electrolytic and non-electrolytic: Electrolytic capacitors hold much more charge;

Electrolytic capacitors - have to be connected with the correct polarity, otherwise they can explode.

Capacitance - is the charge required to cause unit potential difference in a conductor.

1 Farad - is the capacitance of a conductor, which has potential difference of 1 volt when it carries a charge of 1 coulomb.

Time constant - is the product RC (capacitance × Resistance)
Polarization - is the relative displacement of positive and negative charge when an external field is applied. It is given by $P = np$, $p$ is induced atomic dipole moment, $n$ is the number of dipoles per unit volume.

Dielectric constant - is a factor which multiplies the capacitance of a capacitor by a factor $K$. It is independent of the shape and size of the capacitor, but its value varies widely for different materials. It is generally a measure of the extent to which a given material is polarized by external field.

Electric susceptibility - is a parameter which directly relates the polarization of a material to the applied field.

Key Terms

Capacitance, Permittivity, Induced dipole moment
Electrolytic capacitor, dielectric constant, Electric displacement
Dielectric, polarization
Farad, Electric susceptibility

Section Expectation

You will carry out derivation of expression of capacitance for a system of two concentric metal spheres forming a capacitor. In addition you will explain how capacitors store energy and the effect of dielectric on capacitance. Further, derivation for expression for capacitance for combinations of capacitors and using these expressions for calculation will be done.
Introduction to the Activity

This activity is concerned mostly with systems consisting of conductors on which charges can be placed. The activity will further establish that the potential of each conductor is linearly related to the excess charge on it and each of the other conductors.

Detailed Description of the Activity (Main Theoretical Elements)

For each task, you have to read and extract more information from the references and links provided

3.1 Calculating the capacitance

(a) Consider a parallel plate capacitor, each of area A, and separation d. Let the charge density be \( \sigma = \frac{Q}{A} \), where \( Q \) is charge on either plate.

\[
\begin{align*}
\text{Figure 3.1} \\
\text{With the lower plate connected to ground, the charge density on its lower side is close to zero.}
\end{align*}
\]

Working directly from Gauss’ theorem, we find the field (which is uniform between the plates) to be

\[
E = \frac{1}{\varepsilon_0} \sigma \text{ volts/m} \quad (3.1)
\]

Where \( \sigma \) = charge density

Then the potential difference is

\[
V = -\int_0^d E \, dx = \frac{Qd}{\varepsilon_0 A} \text{ volts} \quad (3.2)
\]

\[\Leftrightarrow C = \frac{Q}{V} \frac{A\varepsilon_0}{d} \text{ farads}\]

This shows that capacitance increases linearly with the area of the electrodes (plate) and inversely proportional to the plate separation.
The charge, \( Q \) on a conductor is linearly proportional to its potential \( V \). The proportionality constant is known as the capacitance and is defined as

\[
C \text{ (farads)} = \frac{Q \text{ (Coulombs)}}{V \text{ (volts)}}
\]

The quantities enclosed in the brackets are the respective units. The farad is a very large unit, so we often use microfarads, where \( 1 \text{ F} = 10^6 \mu \text{F} \).

(b) Capacitance between concentric spherical conductors

The capacitance of a spherical capacitor consisting of a spherical conducting shell of radius \( b \) and charge \(-Q\) that is concentric with a smaller conducting sphere of radius \( a \) and charge \(+Q\) is given by

\[
C = \frac{Q}{V} = \frac{ab}{k(b-a)}
\]

(c) Capacitance between two coaxial cylinders of radii \( a \) and \( b \), and length \( L \) is given by

\[
C = \frac{2\pi \varepsilon_0 L}{\ln(b/a)}
\]

where \( a \) and \( b \) are radii of inner and outer cylinders respectively.

### Task 3.2 Derivation of capacitance of capacitors in series and Parallel

The equivalent capacitance, \( C \) of capacitances, \( C \) of capacitances \( C_1, C_2, C_3 \ldots \) of capacitors connected in parallel is given by

\[
C = \sum_i C_i
\]

While the equivalent capacitance, \( C \) of capacitances, \( C \) of capacitances \( C_1, C_2, C_3 \ldots \) of capacitors connected in series is given by

\[
\frac{1}{C} = \sum_i \frac{1}{C_i}
\]

### Task 3.3 Electrostatic stored energy

The work done in the charging process goes into stored energy \( U \) in the system. Thus the potential energy of the charged system is

\[
U = \frac{1}{2} CV^2 \text{ joules}
\]

Where \( C \) is capacitance and \( V \) is the voltage.
Task 3.4 Capacitors with dielectric

When the space between the plates of a capacitor is completely filled with insulating matter, called dielectrics, the capacitance is multiplied by a factor K greater than 1. This factor is called the dielectric constant. The polarization charge $q_p$, the element of surface area $dS$, and surface density of charge $\sigma_p$ are related by the following expression

$$\sigma_p = \frac{dq_p}{dS} \quad (3.6)$$

Capacitance of a capacitor with a dielectric is given by

$$C = \frac{\varepsilon_0 (1 + \chi) A}{d} \quad (3.7)$$

Where $\chi$ is susceptibility, $d$ is in the case of isotropic dielectric, where polarization $P$ is always parallel to the field $E$, the electric displacement, $D$, is given by

$$D = \varepsilon_0 E + P \quad (3.8)$$

Task 3.1.1 Calculation of capacitance

(a) Read the following references: Arthur F Kip (1969); Serway (1986) and Grant (1990) and make notes on potential. As you make notes, provide responses to the following tasks

(b) Use equation (3.1) and show that electric field $E = \frac{Q}{\sigma A}$

(c) Show the steps used in deriving equation (3.2)

(d) Example: Calculation of capacitance of two concentric spherical conductors

If we consider a spherical capacitor consisting of a spherical conducting shell of radius $b$ and charge $-Q$ that is concentric with a smaller conducting sphere of radius $a$ and charge $+Q$. Find its capacitance. if the outer sphere is earthed

You have to note tha:

The field outside a spherical symmetric charge distribution is radial and given by $kQ/r^2$.

In this example, this corresponds to the field between the spheres ($a < r < b$). Since the field is zero elsewhere).

From Gauss’s law we see that only the inner sphere contributes to the field. Thus the potential difference between the spheres is given by

$$V_b - V_a = \int_a^b E \, dr = -kQ \int_a^b \frac{dr}{r^2} = kQ \left[ \frac{1}{r} \right]_a^b = kQ \left( \frac{1}{b} - \frac{1}{a} \right)$$
The magnitude of the potential difference is given by

\[ V = V_a - V_b = kQ \frac{(b-a)}{ab} \]

Substituting this in \[ C = \frac{Q}{V} \], we get

\[ C = \frac{Q}{V} = \frac{ab}{k(b-a)} \]

FOLLOW THE STEPS GIVEN AND SEE THAT THEY ARE CLEAR TO YOU

(e) Calculation of capacitance due two coaxial cylinders

Equation (3.4) gives the capacitance of two coaxial cylinders as

\[ C = \frac{2\pi\varepsilon_0L}{\ln(b/a)} \]

Follow the example given and other literatures in the links and references provided and derive this relation.

Use equations (3.3) and (3.4) for solving different numerical problems

**Task 3.2.1 To derive expression for energy stored**

(a) Make your note and show that equation (3.3) can be written as

\[ U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}Q^2 \frac{Q}{C} \]

**Task 3.3.1 Electrostatic stored energy**

(a) Use the relation \[ Q = CV \] and the definition of work to show that energy \[ U \] stored in capacitor is given by

\[ U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}Q^2 \frac{Q}{C} \] joules

**Task 3.4.1 Dielectrics**

(a) Read and make notes on dielectric materials, dielectric constant, polarization susceptibility, and electric displacement When a charge \( Q \) is placed on the upper isolated plate, all the charge moves to the bottom surface of the plate, and that an equal and opposite charge appears on the lower plate. The equal and opposite charges on the lower plate appear because of the zero-field requirement inside a conductor.

(b) Use the references and the links and derive equations (3.7) and equation (3.8)
(c) Use equations (3.2) and (3.7) to write an expression for the relation between dielectric constant $K$ and susceptibility, $\chi$

**Task 3.5  Experiment on Graphical Representation and Quantitative Treatment of Capacitor Discharge**

You could work in a group to do this experiment

**Apparatus**

- One milliammeter
- One Capacitor 470 $\mu$F
- Assorted resistors
- Stop watch
- Connecting wires
- Reed switch

![Figure 3.2](image)

**Part I**

**Procedure**

(a) Connect switch to point 1 and leave the capacitor to charge
(b) Disconnect the switch from point 1 and connect it to 2
(c) Note the reading on voltmeter and milliammeter
(d) Repeat this at a definite interval of time (say 5s)
(e) Enter your results in a table, including values of charge ($Q$), and time in seconds.
(f) Plot a graph of $Q$ against time

The graph that you get should be similar to that shown in Fig. 3.3

Repeat the experiment with different values of $V$. What do you observe about

(i) Shape of the graph in relation to the voltage?
(ii) The half life of the decay in relation to the voltage?

The expected graph should be asymptotic. In theory what the graph tells us is that the capacitor does not completely discharge, although in practice, it does.
The graph is described by the relationship:

\[ Q = Q_0 e^{-\frac{t}{RC}} \]

Q – charge (C); \( Q_0 \) – charge at the start; \( e \) – exponential number (2.718…); t – time (s); C – capacitance (F); R – resistance (Ω).

**Part II**

Investigate the effects of

I. increasing the resistance on the time taken to discharge the capacitor

II. increasing the capacitance on the time needed to discharge a capacitor

**Formative Evaluation**

Try to discuss some of these evaluation questions with a friend. This practice is very helpful

1. A 5000\( \mu \)F capacitor is charged to 12.0 V and discharged through a 2000 Ωresistor.
   (a) What is the time constant? (Ans. 10 s)
   (b) What is the voltage after 13 s? (Ans. V = 3.3 volts)
   (c) What is the half-life of the decay? (Ans. 6.93 s.)
   (d) How long would it take the capacitor to discharge to 2.0 (Ans. 17.9 s)
2. Write down what is meant by the following terms:
   (a) Dielectric
   (b) Farad
   (c) Working voltage

3. A 470 \( \mu \)F capacitor, charged up to 12.0 V is connected to a 100 kW resistor.
   (a) What is the time constant?
   (b) What is the voltage after 10 s?
   (c) How long does it take for the voltage to drop to 2.0 V?


**Activity 4**

**Title: Direct Current**

You will require 30 hours to complete this activity. Only basic guidelines are provided for you in order to help you do the rest of the curriculum in the activity. Personal reading and work is strongly advised here.

**Specific Objectives**

- Derive the equation for the current density
- Explain the physical basis of Ohm’s law and use Ohm’s law in solving various problem of resistors connected in parallel and in series
- State and use the Kirchoff’s laws in circuit analysis
- Perform mesh analysis of equivalent circuits
- Give definition of resistivity
- Write general expression for resistance which includes the effect of length and cross-section explicitly
- Define, derive and use expressions for maximum power transfer and maximum power transfer efficiency
- Define, derive and use expressions for maximum power transfer efficiency
- Derive expression for torque on current loop and apply the expression to calculate related problems
- Define magnetic dipole
- Write and apply the expression for dipole moment for calculation

**Section Expectation**

Definitions and derivations of expressions will be done. In addition numerical problems will be solved.

**Summary of the Learning Activity**

The expressions including among other things, equations for current density; expression for resistivity; maximum power transfer and maximum power transfer efficiency will be derived. In addition you will perform mesh analysis of equivalent circuit.
Key Concepts

**Current density** - is the rate of charge transport per unit area of cross section. It is defined as a vector $j$, where

$$j = nev \text{ amp/m}^2 \quad (4.1)$$

$v$ is the drift velocity of charge carriers

When the rate of flow of charge varies in a conductor, instead of discussing the current, $I$, we discuss current density $j$ given by:

$$i = \int j \cdot dS \quad (4.2)$$

$dS$ is an element of cross-section area $A$.

**Maximum power (transfer) theorem** - states that to obtain maximum power from a source with a fixed internal resistance the resistance of the load must be made the same as that of the source.

**Current** - is defined as the rate of flow of charge

**Current density** - is the current that flows through a conductor per unit area

![Figure 4.1](image)

With reference to Fig. 4.1, when the current density is uniform, equation (4.2) can be integrated to give

$$i = j \cdot A$$

When the area is taken perpendicular to the current, this equation becomes

$$i = jA$$

where $A$ is the cross-section area of conductor.

**Resistivity** - is resistance of a unit volume of material having unit length and unit cross-section area. It is measured in ohm-metre.
**Key terms**

Conductivity  
Resistivity  
Current  
Current density  
Resistivity

**List of Relevant Useful Links**

http://www.art-sci.udel.edu/ghw/phys245/05S/classpages/mesh-analysis.html 30/08/1006
Introduction to The Activity

This activity turns our attention away from considerations of electrostatic effects to discussion of electric currents, and of the circuits in which the current flows. Also we consider the experimental facts of current flow and the parameters that are useful in the description of currents in the circuits.

Detailed Description of the Activity (Main Theoretical Elements)

4.1 Ohm’s law, series and parallel circuits

Ohm’s law relates three variables: current (I); potential difference (V), and resistance (R). The relation between these three quantities is

\[ V = IR \]  

(4.1)

In a microscopic form, Ohm’s law in terms of the current density, \( j \), field \( E \) in a region as \( j = \sigma E \)

(4.2)

where \( \sigma \) is the conductivity measured in (ohm-metre)\(^{-1}\).

The combined (equivalent) resistance, \( R \), of resistors, \( R_1, R_2, R_3 \ldots \) in series and parallel are given respectively as:

\[ R = \sum R_i \quad \text{For series connection} \]  

(4.3)

\[ \frac{1}{R} = \sum \frac{1}{R_i} \quad \text{For parallel connection} \]  

(4.4)

4.2 Kirchhoff’s Law

Kirchhoff’s Junction Rule

\[ \sum I_{in} = \sum I_{out} \]  

(4.5)

That is, the sum of current, \( I_{in} \) towards a junction equals to the sum of current, \( I_{out} \) leaving the junction.

This rule is equivalent to a statement about conservation of charge.

Kirchhoff’s Loop Rule

\[ \sum \Delta V = 0 \quad \text{For a closed loop} \]  

(4.6)

Each loop should begin and end at the same position in the circuit to be considered closed. The rules for assigning SIGNS to the voltage changes across a battery in a closed loop for Kirchhoff’s loop rule are:

- \( V = -\varepsilon \) if the direction of the loop crosses a battery from positive to negative (high to low)
- \( V = +\varepsilon \) if the direction of the loop crosses a battery from negative to positive (low to high)
Kirchoff’s loop rule is used to determine the correct orientation of batteries in circuits which has more than one battery -- that is, which one(s) are charging and which one(s) are discharging.

This rule is equivalent to a statement about conservation of energy; remember, volt = joule / coulomb

4.3 Maximising power transfer versus power efficiency

To achieve maximum efficiency, the resistance of the source could be made close to zero.

\[ \eta = \frac{R_L}{R_L + R_s} = \frac{1}{1 + \frac{R_s}{R_L}} \]  

(4.7)

\( R_s \) is the source resistance, while \( R_L \) is the load resistance.
Student activity

Task 4.1 Ohm’s law

a. Read the following references: Arthur F Kip (1969); Serway (1986), and Grant (1990) and make notes on electric current. Make notes for Ohm’s law; concepts behind Kischoff’s laws; resistivity; power transfer efficiency. provide responses to the tasks that you will be asked to deal with under student’s activity. Use also the links provided.
b. Use the definition of current density and write out its SI unit.
c. (i) You should be able to show that in a circuit where an external resistor, R is connected to a cell of e.m.f, E and internal resistance, r, the three are related by 
\[ E = IR + Ir \]
Describe all the steps clearly.
(ii) Now show that \[ IE = FR + Fr \]
Hint: Use equation (4.1) and make I the subject.
d. If n is the number of carriers per unit volume, e the electric charge and \( \nu \) is drift velocity show that current density \( j = ne\nu \)

Task 4.2 Example on Ohm’s law

Study the following steps required in deriving the microscopic form of Ohm’s law.

Consider charge carrier drifting with mean velocity \( \nu \) through a conductor of cross-section area A and assume that the density of the carrier is n per unit volume.

- In one second the carriers occupy a volume of \( V = \nu A \)
- Total charge in this volume = neA \( e \) is charge on each carrier
- But \( i = \) charge per second
- Thus \( i = ne\nu A \)
Recall that \( j = i/A \)
- Therefore \( j = ne\nu \)

When we consider effects of length, \( L \), and cross-section area, \( A \), a general expression for resistance is
\[ R = \rho \frac{L}{A}, \quad \rho \text{ is resistivity.} \]
If the electric field which causes the current flow is $E$ and we characterize current by $j$, we will be able to describe local behavior in any microscopic region. Thus, the microscopic form of Ohm’s law is obtained as follows:

Recall that

$$V = iR$$

$$V = EL, \quad R = \frac{\rho L}{A}; \quad and \quad j = \frac{i}{A}$$

Substituting these in equation (ii) we have

$$j = \frac{1}{\rho} E$$

Therefore

$$j = \sigma E$$

- Check and show that $\sigma = \frac{1}{\rho}$, where $\sigma$ is conductivity

**Task 4.2.1 Kirchoff’s law**

(a) You need to consult the references and the links and make notes.

(b) Apply the two laws of Kirchoff to Fig 4.2 and write both Kirchoff’s loop rule and Kirchoff’s junction rule

![Figure 4.2](image)

*Hint.* The direction indicated on the loop should guide you
Task 4.3.1 Maximum power efficiency

(a) Define efficiency
(b) Using the definition of efficiency, derive expression in equation (4.7)
   (i) If $R_L = R_s$ then $\eta = 0.5$
   (ii) If $R_L = \infty$ then $\eta = 1$
   (iii) If $R_L = 0$ then $\eta = 0$

These three examples show you that:
The efficiency is only 50% when maximum power transfer is achieved, but approaches 100% as the load resistance approaches infinity (though the total power level tends towards zero). When the load resistance is zero, all the power is consumed inside the source (the power dissipated in a short circuit is zero) so the efficiency is zero.

Task 4.4 Mesh analysis

Objectives:
1. To construct a planar circuit having two voltage sources and five resistors.
2. To study node voltages and mesh currents.
3. To compare calculated and measured results using both nodal and mesh analysis.

Equipment and Parts List:
- DC Power Supply.
- Digital Multimeter (DMM)
- Resistors one each: 1.5 kW, 2.2 kW, 4.7 kW, 5.6 kW, and 6.8 kW.
- Breadboard

Procedure:
1. Connect the circuit shown in Fig. 4.3. VS1 and VS2 are DC power supply.
   $R_1 = 2.2 \text{k}\Omega$, $R_2 = 4.7 \text{k}\Omega$, $R_3 = 6.8 \text{k}\Omega$, $R_4 = 5.6 \text{k}\Omega$, $R_5 = 1.5 \text{k}\Omega$.

Figure 4.3
2. Set $V_{S1} = 12 \, \text{V}$ and $V_{S2} = -12 \, \text{V}$. Note that Node 1 is positive and Node 4 is negative.
3. Measure and Record the readings corresponding to $V_1$, $V_2$, $V_3$, and $V_4$. Similarly record also the current $I_1$, $I_2$, and $I_3$. (not branch current $I_a$ and $I_b$)
4. Use a multimeter and measure the values of all resistors.

**Comparisons And Questions:**

1. From your measured mesh currents, calculate the value of the branch currents $I_a$ and $I_b$ shown in Figure 4.3.
2. By observation, what are the values of $V_1$ and $V_4$? With the given values of $V_{S1}$ and $V_{S2}$.
3. Node Equations:
   (a). Set up the nodal equations for the circuit, and solve for $V_2$ and $V_3$, using nominal values of resistances and nominal voltage sources.
   (b). Compare all measured node voltages with the calculated values.
   (c). Repeat a & b using the measured values of resistances and measured values of the source voltages.
4. Mesh Equations:
   (a). Set up the mesh equations for the circuit, and solve for the three mesh currents, using nominal values of resistances and the nominal voltage sources.
   (b). Compare all measured mesh currents with the calculated values.
   (c). Repeat a & b using the measured values of resistance and measured values of the source voltages.
5. Calculate the power absorbed by resistors $R_2$ and $R_4$. For each resistor calculate power by using three different methods: $P=VI$, $P=IR$, $P=V^2/R$. Use measured resistances, measured node voltages, and the branch currents calculated from the measured mesh currents. Explain any differences in the power obtained by the three methods.

**Conclusions:**

Based on your experimental observations, what laws and principles have been verified by this experiment?
Formative Evaluation

1. The multiloop circuit shown in Fig. 4.4 contains three resistors, three batteries, and one capacitor under steady-state conditions, find the unknown currents
   (a) Show that the charge on the capacitor is 66.0 µC
   (b) Why is the left side of the capacitor positively charged
   (c) Show that the voltage across the capacitor by traversing any other loop, such as the outside loop (Answer = 11.0V)

2. Write up Kirchoff’s junction rules for Fig. 4.5

Figure 4.4

Figure 4.5
**Activity 5**

**Title: Magnetism**

You will require 30 hours to complete this activity. Only basic guidelines are provided for you in order to help you do the rest of the curriculum in the activity. Personal reading and work is strongly advised here.

**Specific Objectives**

- Define the terms: magnetic field, magnetic flux and flux density
- Explain and draw magnetic field lines associated with current carrying conductors, and explain the principles of instruments based in it;
- Explain the principles of an oscilloscope;
- State, explain and use Faraday’s law of electromagnetic induction;
- Derive expression for force on a current-carrying wire in a magnetic field
- Relate the force (F) to velocity (v), charge (q) and magnetic field (B)
- Demonstrate magnetic field and interaction using magnets, and current-carrying wire, show the influence of the magnetic field by a moving charge using an oscilloscope, and demonstrate the electromagnetic induction/Faraday’s law using simple materials
- Derive expression for torque on current loop and apply the expression to calculate related problems
- Define magnetic dipole
- Write and apply the expression for dipole moment for calculation

**Section Expectation**

- Derivation and explanation of Faraday’s laws of electromagnetic induction
- Derive expression for force on current-carrying wire in magnetic field.
- Relating the force to velocity, charge and magnetic field.
- Derivation of expression for torque, magnetic fields in solenoid and toroids.
- Defining magnetic filed, magnetic flux; magnetic dipole
- Stating and using Ampere’s law

**Summary of the Learning Activity**

Definitions of a number of terms including: magnetic field, magnetic flux and flux density. In addition explanation of a number of concepts like Motion of a charged particle in a magnetic field and Magnetic moment, \( \mu \), of a coil. It includes also stating of laws and principles and their applications.
**Keys concepts**

**Magnetic flux** through an are element \( dA \) is given by \( B \cdot dA = B dA \cos \theta \). \( dA \) is perpendicular to the surface.

**Magnetic moment of a current loop** carrying current, \( I \) - is

\[ \mu = IA \]

Where \( A \) is perpendicular to the plane of the loop.

**Motion of a charged particle in a magnetic field** - The magnetic force acting on a charged particle moving in a magnetic field is always perpendicular to the velocity of the particle. Thus, the work done by the magnetic force is zero since the displacement of the charge is always perpendicular to the magnetic force. Therefore, a static magnetic field changes the direction of the velocity but does not affect the speed of kinetic energy of the charged particle.

**Magnetic moment, \( m \)** of a coil - Is the combination \( NIA \), \( N \) is number of turns, \( I \) is current and \( A \) is area of coil

**Magnetic dipole moment** \( \mu \), of a small coil of area \( S \) carrying a current \( I \) is defined as

\[ \mu = IS \]

where \( S \) is the vector perpendicular to the plane of the coil in the direction related to the current by the right-hand rule.

**Key Terms**

- Magnetic dipole
- Magnetic field
- Magnetic force
- Magnetic flux
- Magnetic moment
- Solenoids
- Torque

**List of Relevant Useful Links**


http://www4.ncsu.edu/~mowat/H&M_WebSite/FaradaysLaw/FaradaysLaw.html 30i08/2006
Introduction to the Activity

Historically, the study of magnetism began with the observations of interactions between ferromagnetic materials; substances such as iron under appropriate conditions exhibit strong forces of attraction and repulsion, which resemble, but are quite distinct from, electrostatic forces. In 1819, Oersted first showed a connection between electricity and magnetism by demonstrating the torque on a compass needle caused by a nearby electric current. Thus in this activity we will first discuss magnetism in terms of the forces between the moving charges of current elements.

In the early 1830s Michael Faraday made the observation that a changing current in one electric circuit can cause current to appear (“induce a current”) in a second circuit. Electromagnetic induction is the principle governing the operation of electric motors, generators, transformers, and some types of particle accelerators. Faraday’s Law is one of the four cornerstones of electromagnetic theory. Without it, we couldn’t have light.

http://www4.ncsu.edu/~mowat/H&M_WebSite/FaradaysLaw/FaradaysLaw.html
30/08/2006

Detailed Description of the Activity
(Main Theoretical Elements)

5.1 Magnetic field, magnetic flux and flux density

(a) Read the following references: Arthur F Kip (1969); Serway (1986), and Grant (1990) and make notes on electric current. Make notes for Magnetism according topics presented below. You are advised to work through each task at a time.

(b) Magnetic flux, field B and cross sectional area through which the flux passes are related by

\[
\phi = BA = \mu_0 \frac{N A}{L} 
\]

The SI unit of flux is the Weber (Wb), while the unit of magnetic field is the Tesla (T)

1 Wb m⁻² = 1 T

5.2 Magnetic force on a current-carrying wire

(a) The total magnetic field \( B \) due to a thin, straight wire carrying a constant current \( I \) at a point \( P \) a distance \( a \) from the wire is

\[
B = \frac{\mu_0 I}{2\pi a} 
\]
If two parallel conductors carry a steady current $I_1$ and $I_2$ respectively and if the separation between them is $a$, then force, $F_1$ on the first conductor per unit length is,

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a} \quad (5.3)$$

### 5.3 Moving Charge in a magnetic field

The relation between force ($F$), velocity ($v$) and charge, ($q$) moving in a conductor placed in a field $B$ is given by:

$$\vec{F} = \frac{\mu_0 q \vec{v} \times \vec{r}}{r^2} \quad (5.4)$$

$$\sigma_0 = 4\pi \times 10^{-7} \ \text{Ns}^2/C^2$$

is called the permeability of free space. The constant $\sigma_0$ that is used in electric field calculations is called the permittivity of free space.

### 5.4 Faraday’s law of electromagnetic induction

(a) Consider a circuit in which there are two circuits: primary circuit, and secondary circuit with $N_s$ turns. When a varying current is fed in the primary circuit an emf $E$ is induced in the secondary coil. The induced emf is given by:

$$E = -\frac{d}{dt}(N_s \vec{B} \cdot \vec{A}) \quad (5.5)$$

(b) Effective area of the secondary coil

By Ohm’s law if the secondary has resistance $R_s$, then the induced current is $I_s = \text{emf}/R_s \quad (5.6)$

In practice, it is not accurate to assume that each turn has the same area $A$. Therefore we will represent the dot product as

$$N_s \vec{B} \cdot \vec{A} = N_s \vec{B} \cdot \vec{A}_{\text{eff}} \cos \theta \quad (5.7)$$

where $\theta$ is the angle between $\vec{B}$ and area vector $\vec{A}$. the effective area is

$$A_{\text{eff}} = \left( \frac{\pi}{3} (r_1^2 + r_2^2 + r_1 r_2) \right) \quad (5.8)$$

where $r_1$ and $r_2$ are the inner and outer radii, respectively, of the secondary coil.

### 5.4 Torque on a current loop

(a) The torque on a coil with $N$ turns of area $A$ carrying a current $I$ is given by:

$$\text{Torque on coil} = \tau = NIBA \sin \phi \quad (5.9)$$
is the angle between the magnetic field and the normal to the plane of the coil. The combination NIA is usually referred to as the magnetic moment, $m$ of the coil. It is a vector normal (i.e., perpendicular) to the loop. If you curl your fingers in the direction of the current around the loop, your thumb will point in the direction of the magnetic moment. Thus

$$\tau = \mu \times B \quad (5.10)$$

Note that this is analogous to the torque acting on an electric dipole moment $p$ in the presence of an external electric field $E$, where $\tau = p \times E$. The SI unit of magnetic moment is ampere-metre$^2$ (A-m$^2$)

**Students Activity**

Read the following references: Arthur F Kip (1969); Serway (1986) and Grant (1990), plus the links provided, and make notes on Magnetic field, magnetic flux and flux density, Magnetic force on a current-carrying wire, Faraday’s law of electromagnetic induction, Torque on a current loop, and Ampere’s circuital law

**Task 5.1 Magnetic field, magnetic flux and flux density**

(a) In your notes, draw and describe the magnetic field lines associated with current carrying conductors

**Task 5.2 Magnetic force on a current-carrying wire**

(a) Make notes and show how equation (5.2) is derived
(b) Apply equation (5.2) to solve numerical problems

**Task 5.3 Moving Charge in a magnetic field**

(a) Study equation (5.4) critically and show how it is derived
(b) Apply equation (5.4) to solve numerical problems

**Task 5.4 Faraday’s law of electromagnetic induction**

(a) State Faraday’s law given in equation (5.5) in words
(b) What is the significance of the minus in equation (5.5)?
(c) Use standard text and the links to show that the magnetic field at the center of the primary coil is approximated by the field at the center of a single circular current, multiplied by the number of turns $N_p$ in the primary: i.e.

$$B = N_p \left( \frac{\mu I_p}{2r} \right) \quad (5.11)$$

(d) What is the significance of torque on a loop?
Task 5.5. **Ampere’s circuital law**

Apply Ampere’s circuital law and derive expressions for magnetic fields in solenoids and toroids. (use the references and the Links)

Task 5.6. **Experiment: Measuring Induced emf**

**Purpose**
- To verify Faraday’s law of electromagnetic induction

**Apparatus**
- The Field Coil made from 200 turns of copper SW.G 28 wire (diameter 0.2684 cm),
- Search Coil is made from 4000 turns of copper S.W.G 36 wire (diameter 0.134 cm).
- A two-channel oscilloscope
- Cables and plugs
- An iron bar long enough to stick through both coils simultaneously.
- (Optional)
- Cables and plugs
- A signal-generator

![Figure 5.1](image)

**Figure 5.1**
Procedure

Follow the steps given below to do the experiment

(a) Place the field coil on a piece of paper and trace around the inside. Find the centre of this tracing and mark it with a cross. This marks the spot to which you will align the search coil.

(b) Measure the inner ($r_1$) and the outer ($r_2$) radii of the search coil and determine its effective area. Measure the inner and outer radii of the field coil and determine its average radius, $r = \frac{r_1^2 + r_2^2 + r_{12}}{3}$

(c) Connect the two coils, the signal and the oscilloscope as shown in Fig. 5.1

(d) Place the field coil on the paper so that its inner diameter coincides with the tracing

(e) Place the search coil such that its centre also coincides with mark at the centre of the field coil.

(f) Turn on the signal generator and oscilloscope

(g) Set the oscilloscope to display both channels 1 and 2

(h) Set the signal generator to produce a triangle wave form of amplitude 8 volts and frequency 400Hz. Use the knobs on the signal to adjust the voltage and frequency. Read their values from the oscilloscope screen, but not from the setting of the signal generator knobs. Set the oscilloscope time scale so you can display three periods of the triangle wave. Set the gains and positions of the two traces so that the triangle wave form fills the top half of the screen and the induced emf wave form fills the lower half of the screen.

(i) Sketch the patterns on the screen indicating the actual settings of the vertical (V/div) and horizontal sweep rate (ms/div)

(j) Read from the oscilloscope the value of dV/dt (read from channel 1) and the measured induced emf, $E_2$ (read from channel 2). Note that

Induced emf $E_1 = -\frac{d}{dt}(N_r B A)$

$I_r = V/R_p$ and [why is this so?]

$B = N_r \frac{\mu_0 I_r}{2r}$ $r$ is the mean value of $r_1$ and $r_2$

Use these equations to write the expression for induced e.m.f. $E_1$

(k) From your measurements evaluate the value of $E_1$ and compare it with $E_2$

(l) Repeat procedure (h) – (j) for different values of input voltage and frequency

(m) Comment of your results
1. A rectangular coil of dimensions 5.40 cm x 8.50 cm consists of 25 turns of wire. The coil carries a current of 15 mA. Calculate the magnitude of the magnetic moment of the coil. (Ans. $1.72 \times 10^{-3}$ A.m²)

Suppose a magnetic field of magnitude of 0.350 T is applied parallel to the plane of the loop. What is the magnitude of the torque acting on the loop? (Ans. $6.02 \times 10^{-4}$ N.m)

2. A proton moves with a speed of $8.0 \times 10^6$ ms⁻¹ along the x-axis. It enters a region where there is a field of magnitude 2.5 T, directed at an angle of 60° to the x-axis and lying in the xy plane. Calculate the initial magnetic force and acceleration of the proton. (Ans. $2.77 \times 10^{-12}$ N; $1.66 \times 10^{13}$ ms⁻²)

3. A proton is moving in a circular orbit of radius 14 cm in a uniform magnetic field of magnitude 0.35 T directed perpendicular to the velocity of the proton. Find the orbital speed of the proton. (Ans. 4.69 ms⁻¹)
XIV. Synthesis Of The Module

Electricity and magnetism I

Activity 1

In activity one, the key concepts are about interactions between charged bodies which ultimately led to Coulomb’s law, hence Gauss’ theorem. The derivations and applications of related expressions are important to master. The starting point to remember is that electric charge is an attribute of matter that produces a force, just as mass causes the gravitational force, but unlike mass, electric charge can be either positive or negative. That, the density of electric lines in a given region is proportional to the magnitude of the field in that region and that the number of electric lines originating or ending on charges is proportional to the magnitude of each charge.

In addition to the physics learnt a number of expressions that have been derived and used need to be learned and put in practice. These include expressions for net torque on a dipole in and external field, that is,

$$\tau = 2aqE \sin \theta = pE \sin \theta$$

By Coulomb’s law force between two charges \(Q_1\) and \(Q_2\) which are at a separation \(r\) apart. is

$$F = \frac{q_1q_2}{4\pi \varepsilon_0 r^2} = \frac{1}{2} \frac{q_1q_2}{r^2} K$$

From definition of electric field, the value of \(E\) due to \(n\) discrete charges \(q_1, \; q_2, \; q_3, \; \ldots q_i, \; \ldots q_n\) at rest by the principle of superposition is

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \sum q_i \frac{1}{r_i^2} \hat{r}$$

Likewise, for a body of continuous charge, the electric field at a distance \(r\) away is

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{dq}{r^2} \hat{r}$$

For a dipole, the dipole field at points in equatorial plane, a distance \(r\), from the centre is given by:

$$E = \frac{p}{4\pi \varepsilon_0 (a^2 + r^2)}$$ Newton/Coulomb
Gauss’ flux theorem which is a direct consequence of Coulomb’s law’s inverse-square law, mathematically is stated as follows:

$$\oiint_{\text{Closed Surface}} E \cos \theta \, ds = \sum_i \frac{q_i}{\varepsilon_0}$$

That is, the surface integral of the normal component of $E$ over a closed surface equals the sum of charges inside the enclosed volume divided by $\varepsilon_0$.

This law is applied in different situations which you need to understand.

**Activity 2**

In activity two the key concepts include electric field, potential and the relation between them. Different expressions linked them are derived and applied. For example:

Electric potential energy (PE) = $qEd$, where $q$ is the charge on an object, $E$ is electric field produced by $Q$, and $d$ is the distance between the two charges.

An analogy between gravitational and electric potential is essential. It is important to know how to derive and apply expressions such as:

Electrical potential, $V = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

where $q_1$ and $q_2$ are the charges at a separation $r$.

Potential and electric are related as follows:

$$E = -\frac{dV}{dx},$$

The general expression for the potential at a given point in space due to a distribution of point charges is

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

And if the distribution is continuous, the expression for potential in terms of volume density of charge $\rho$ which may vary from point to point is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{\rho dV}{r}$$

Mastery of how to derive and apply expressions for potential and electric field due to a dipole at a point $P$ is essential. For example:

$$V = \frac{1}{4\pi\varepsilon_0} \frac{\rho \cos \theta}{r^2}$$

and

Electric field is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{\rho}{r} \left(3 \cos^2 \theta + 1\right)^{\frac{1}{2}}$$
This should also include expressions for potential and electric field at P along the axis of a uniformly charged ring should

**Activity 3**

The use of Gauss’ theorem is an important tool for deriving expression of electric field across parallel plates capacitor, that is,

\[ E = \frac{1}{\varepsilon_0} \sigma \text{ volts/m} \]

Where \( \sigma \) = charge density

This subsequently leads to expression for potential difference as

\[ V = -\int_{0}^{d} E \, dx = \frac{Qd}{\varepsilon_0 A} \text{ volts} \]

\[ \Leftrightarrow C = \frac{Q}{V} = \frac{A\varepsilon_0}{d} \text{ farads} \]

For good practice, one needs to be able to show how capacitance between concentric spherical conductors consisting of a spherical conducting shell of radius \( b \) and charge \(-Q\), that is concentric with a smaller conducting sphere of radius \( a \) and charge \(+Q\) being given by.

\[ C = \frac{Q}{V} = \frac{ab}{k(b-a)} \]

In addition, one needs to be able also to derive expression for capacitance between two coaxial cylinders of radii \( a \) and \( b \), and length \( L \) as

\[ C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \]

where \( a \) and \( b \) are radii of inner and outer cylinders respectively.

When capacitors of capacitances \( C_1, C_2, C_3 \ldots \) are connected in parallel, their equivalent capacitance, \( C \) is given by

\[ C = \sum_i C_i \]

While, if they are series, the equivalent capacitance, \( C \) of capacitances is given by

\[ \frac{1}{C} = \sum_i \frac{1}{C_i} \]
There are other important relations derived in this section that one needs to be familiar with. These include expression for capacitance of a capacitor with a dielectric being given as

$$ C = \frac{\varepsilon_0 (1 + \chi) A}{d} $$

Where $\chi$ is susceptability, $d$ is separation between the plates.

**Activity 4**

Activity four a number of relations have been derived. These include

(i) A microscopic form of Ohm’s law in terms of the current density, $j$, field $E$ in the region, whereby

$$ j = \sigma E $$

$\sigma$ is the conductivity measured in (ohm-metre)$^{-1}$.

(ii) The equivalent resistance, $R$, of resistors, $R_1, R_2, R_3…$ in series and parallel are given respectively as:

For series connection

$$ R = \sum R_i $$

For parallel connection

$$ \frac{1}{R} = \sum \frac{1}{R_i} $$

Other important relations are Kirchoff’s junction laws. These are

$$ \sum I_{in} = \sum I_{out} $$

That is, the sum of current, $I_{in}$ towards a junction equals to the sum of current, $I_{out}$ leaving the junction and,

$$ \sum \Delta V = 0 $$

For a closed loop

It is also important to learn how to derive expression for maximum efficiency for system of resistors connected to a power source.

**Activity 5**

In activity five there a number of expressions that you need to be acquainted with. The important expressions includes

(a) Magnetic flux, field $B$ and cross sectional area through which the flux passes are related by

$$ \phi = BA = \mu_0 \frac{N_i A}{L} $$
(b) The total magnetic field $\mathbf{B}$ due to a thin, straight wire carrying a constant current $I$ at a point $P$ a distance $a$ from the wire is

$$\mathbf{B} = \frac{\mu_0 I}{2\pi a}$$

(c) If two parallel conductors carry a steady current $I_1$ and $I_2$ respectively and if the separation between them is $a$, then force, $F_1$ on the first conductor per unit length is,

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a}$$

(d) The relation between force ($F$), velocity ($v$) and charge, ($q$) moving in a conductor placed in a field $\mathbf{B}$ is given by:

$$\vec{F} = \frac{\mu_0 I}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$\sigma_0 = 4\pi \times 10^{-7} \text{ Ns}^2 \text{ C}^{-2}$$

is called the permeability of free space. The constant $\epsilon_0$ that is used in electric field calculations is called the permittivity of free space.

(e) Faraday’s law of electromagnetic induction, in which we consider two circuit: primary circuit, and secondary circuit with $N_s$ turns. When a varying current is fed in the primary circuit an emf $E$ is induced in the secondary coil. The induced emf is given by:

$$E = -\frac{d}{dt}(N_s \mathbf{B} \cdot \mathbf{A})$$

**Expected Solutions To Some Problems Set**

**Solution to 1.5.1**

(a) The outward flux of $\mathbf{E}$ over any closed surface is equal to the algebraic sum of the charges enclosed by the surface divided by $\epsilon_0$. 


2.2.1

Since average static field inside a metal is zero, the charge resides on the surface. A spherical charge conductor behaves as though all charges are concentrated at its centre. This allows us to use expression for field of a point charge

Thus,

\[ V_r = \int E \cdot dr = -\frac{1}{4\pi \varepsilon_0} \int \frac{Q}{r^2} dr = \frac{Q}{4\pi \varepsilon_0 r_0} \]

The assumption is that \( V = 0 \) at infinity. The similarity with a point charge is due to identical field distributions.

2.3.1

Electric potential, \( V \), and electric potential energy, \( U = qV \), are different quantities, with different dimensions and different SI units. Energy of any kind (electrical \( qV \), gravitational \( mgh \), etc) represents the same physical quantity. Electrical potential, \( V \), is the equivalent of “height”/”level”/”altitude”, \( h \) in the case of gravitational field. Therefore, electric potential, \( V \), and electric potential energy, \( qV \), are as different as @height@, \( h \), is different from potential gravitational energy, \( U = mgh \).

2.3.2

Application of the concepts Example: Potential due to a uniformly charged ring

Let us take point \( P \) at a distance \( x \) from the centre of the ring. The charge element \( dq \) is at a distance \( \sqrt{x^2 + a^2} \) from point \( P \).

We can write

\[ V = k \int \frac{dq}{r} = k \int \frac{dq}{\sqrt{x^2 + a^2}} = \frac{k}{\sqrt{x^2 + a^2}} \int dq = \frac{kQ}{\sqrt{x^2 + a^2}} \]

\[ k = \frac{1}{4\pi \varepsilon_0} \]

In this case each element \( dq \) is the same distance from \( P \).
The electric field $\mathbf{E}$

From the symmetry, we see that along the $x$ axis $\mathbf{E}$ can have only an $x$ component. Therefore, we can use the expression

$$E_x = \frac{dv}{dx}, \text{ (Show that this relation is true), to find the electric field at P}$$

$$E_x = - \frac{dV}{dx} = - kQ \frac{d}{dx} \left( x^2 + a^2 \right)^{1/2}$$
XV. Summative Evaluation

1. Use the diagram given above and show that potential, V, and electric field, E, due to a dipole at point P are respectively given by
   
   (i) \[ V = \frac{1}{4\pi \varepsilon_0} \frac{p \cos \theta}{r^2} \]
   and
   
   (ii) \[ E = \frac{1}{4\pi \varepsilon_0} \frac{p}{r^3} \left( 3 \cos^2 \theta + 1 \right)^{\frac{3}{2}} \]

   (iii) Two point charges +Q and –Q are placed at a distance 2a apart. Show that the dipole field at points in equatorial plane, a distance r, from the centre is given by
   
   \[ E = \frac{p}{4\pi \varepsilon_0} \frac{1}{(a^2 + r^2)^{\frac{3}{2}}} \] \text{ Newton/coulomb}

2. (a) Show that in a circuit where an external resistor, R is connected to a cell of e.m.f, E and internal resistance, r, the three are related by
   
   \[ E = IR + Ir \]

   (b) Hence show that \[ IE = E^2R + Fr. \]

   (c) If n is the number of carriers per unit volume, e the electric charge and v is drift velocity show that current density
   
   \[ j = nev \]
3. The square of side a above contains a positive charge –Q fixed at the lower left corner and positive point charges +Q fixed at the other three corners of the square. Point S is located at the centre of the square.

   (a) On the diagram, indicate with an arrow the direction of the net electric field at point S.

   (b) Derive expressions for each of the following in terms of the given quantities and fundamental constants

      (i) The magnitude of the electric field at point S

      (ii) The electric potential at point S.

4. The circuit above contains a capacitor of capacitance C, power supply of emf E, two resistors of resistances R₁ and R₂, and two switches K₁ and K₂. Initially, the capacitor is uncharged and both switches are open. Switch K₁ is closed at time t = 0.

   (a) write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

   (b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t, if E = 6V, C = 0.030 F, R₁ = R₂ = 5000 Ω.

   (c) Determine the time when the capacitor has a voltage of 2.0 V across it.

   (d) If after a long time, switch K₂ is closed at a new time t = 0, sketch graphs of the current I₁ in R₁ versus time, and the current I₂ in R₂ versus time, starting when K₂ is closed at a new time t = 0.
5. Show that

(a) the magnetic flux, \( \Phi \), field, \( B \) and cross sectional area through which the flux passes are related by
\[
\Phi = \mu_0 \frac{N_1 A}{L}
\]

(b) the total magnetic field \( B \) due to a thin, straight wire carrying a constant current \( I \) at a point \( P \) a distance \( a \) from the wire is
\[
B = \frac{\mu_0 I}{2\pi a}
\]

(c) If two parallel conductors carry a steady current \( I_1 \) and \( I_2 \) respectively and if the separation between them is \( a \), show that the force, \( F_1 \) on the first conductor per unit length is,
\[
F_1 = \frac{\mu_0 I_1 I_2}{2\pi a}
\]

6.

A rectangular loop of dimensions \( 3l \) and \( 4l \) lies in the plane of the page as shown above. A long straight wire also in the plane of the page carries a current \( I \).

(a) Calculate the magnetic flux through the rectangular loop in terms of \( I \), \( l \), and any relevant constants.

(b) Beginning with time \( t = 0 \), the current in the long straight wire is given as a function of time \( t \) by
\[
I(t) = I_0 e^{-kt}
\]
\( I_0 \) and \( k \) are constants
If the loop has a resistance \( R \), calculate in terms of \( R \), \( I_0 \), \( l \), \( k \), and the fundamental constants
(i) the current in the loop as a function of time \( t \)
(ii) the total energy dissipated in the loop from \( t = 0 \) to \( t = \infty \)
7. A spherical cloud of charge of radius $R$ contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \quad \text{for} \quad r \leq R$$

$$\rho = 0 \quad \text{for} \quad r > R$$

Where $r$ is the distance from the centre of the cloud. Express all algebraic answers in terms of $Q$, $R$, and fundamental constants.

(a) Determine the following as a function of $r$ for $r > R$.

(i) the magnitude of $E$ of the electric field

(ii) the electric potential.

(b) A proton is placed at a point $P$ shown above and released. Describe its motion for a long time after its release.

8. A circular wire loop with radius $0.02 \, \text{m}$ and resistance $150 \, \Omega$ is suspended horizontally in a magnetic field of magnitude $B$ directed upward at an angle $70^\circ$ with the vertical as shown above. The magnitude of the field is given as a function of time $t$ in seconds as

$$B = 8(1 - 0.1t)$$

(a) Determine the magnetix flux $\Phi_m$ through the loop as a function of time

(b) Sketch the magnetic flux $\Phi_m$ as a function of time.

(c) Determine the magnitude of the induced emf in the loop

(d) Determine the magnitude of the induced current in the loop
9. (a) Show that the capacitance of a spherical capacitor consisting of a spherical conducting shell of radius $b$ and charge $-Q$ that is concentric with smaller conducting sphere of radius $a$ and charge $+Q$ is given by:

$$C = \frac{Q}{V} = \frac{ab}{k(b-a)}$$

(b) Show that the capacitance between two coaxial cylinders of radii $a$ and $b$, and length $L$ is given by:

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

where $a$ and $b$ are radii of inner and outer cylinders respectively.

10. The multiloop circuit shown above contains three resistors, three batteries, and one capacitor under steady-state conditions, find the unknown currents:

(a) Calculate the magnitude of the charge on the capacitor.

(b) Calculate the pd across the 4Ω resistor.
XVI. REFERENCES

List of Relevant Readings FOR ALL ACTIVITIES


Kathryn Whyman (2003). Electricity and magnetism


XVII. Main Author of The Module

Dr Sam Kinyera Obwoya was born on September 05, 1954 in Gulu District in Uganda. He is a teacher trainer at Kyambogo University, Uganda, and a materials scientist. He is a Senior Lecturer and the Director of Open Distance and e-Learning Centre (ODeL). He has published many papers in Materials Science and Physics Education. Currently, he is a fellow of US/Africa Materials Science Institute (USAMI), Princeton University, USA where he is a Visiting Scientist.