Education: Mathematics MAT02

BASIC MATHEMATICS

Jairus M. Khalagai
The African Virtual University (AVU) is proud to participate in increasing access to education in African countries through the production of quality learning materials. We are also proud to contribute to global knowledge as our Open Educational Resources (OERs) are mostly accessed from outside the African continent. This module was prepared in collaboration with twenty one (21) African partner institutions which participated in the AVU Multinational Project I and II.

From 2005 to 2011, an ICT-integrated Teacher Education Program, funded by the African Development Bank, was developed and offered by 12 universities drawn from 10 countries which worked collaboratively to design, develop, and deliver their own Open Distance and e-Learning (ODeL) programs for teachers in Biology, Chemistry, Physics, Math, ICTs for teachers, and Teacher Education Professional Development. Four Bachelors of Education in mathematics and sciences were developed and peer-reviewed by African Subject Matter Experts (SMEs) from the participating institutions. A total of 73 modules were developed and translated to ensure availability in English, French and Portuguese making it a total of 219 modules. These modules have also been made available as Open Educational Resources (OER) on oer.avu.org, and have since then been accessed over 2 million times.

In 2012 a second phase of this project was launched to build on the existing teacher education modules, learning from the lessons of the existing teacher education program, reviewing the existing modules and creating new ones. This exercise was completed in 2017.

On behalf of the African Virtual University and our patron, our partner institutions, the African Development Bank, I invite you to use this module in your institution, for your own education, to share it as widely as possible, and to participate actively in the AVU communities of practice of your interest. We are committed to be on the frontline of developing and sharing open educational resources.

The African Virtual University (AVU) is a Pan African Intergovernmental Organization established by charter with the mandate of significantly increasing access to quality higher education and training through the innovative use of information communication technologies. A Charter, establishing the AVU as an Intergovernmental Organization, has been signed so far by nineteen (19) African Governments - Kenya, Senegal, Mauritania, Mali, Cote d’Ivoire, Tanzania, Mozambique, Democratic Republic of Congo, Benin, Ghana, Republic of Guinea, Burkina Faso, Niger, South Sudan, Sudan, The Gambia, Guinea-Bissau, Ethiopia and Cape Verde.

The following institutions participated in the teacher education program of the Multinational Project I: University of Nairobi – Kenya, Kyambogo University – Uganda, Open University of Tanzania, University of Zambia, University of Zimbabwe – Zimbabwe, Jimma University – Ethiopia, Amoud University - Somalia; Université Cheikh Anta Diop (UCAD)-Senegal, Université d’ Antananarivo – Madagascar, Universidade Pedagogica – Mozambique, East African University - Somalia, and University of Hargeisa - Somalia.
The following institutions participated in the teacher education program of the Multinational Project II: University of Juba (UOJ) - South Sudan, University of The Gambia (UTG), University of Port Harcourt (UNIPORT) – Nigeria, Open University of Sudan (OUS) – Sudan, University of Education Winneba (UEW) – Ghana, University of Cape Verde (UniCV) – Cape Verde, Institut des Sciences (IDS) – Burkina Faso, Ecole Normale Supérieure (ENSUP) - Mali, Université Abdou Moumouni (UAM) - Niger, Institut Supérieur Pédagogique de la Gombe (ISPG) – Democratic Republic of Congo and Escola Normal Superieur Tchicote – Guinea Bissau

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The Rector
African Virtual University
Production Credits

This second edition is the result of the revision of the first edition of this module. The informations provided below, at the exception of the name of the author of the first edition, refer to the second edition.

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Basic Mathematics

Table of Contents

Foreword ................................................. 2
Production Credits .................................... 3
Copyright Notice ...................................... 4
Supported By ........................................... 4
Introduction ........................................... 7

Title of Module. ....................................... 7
Prerequisite Courses or Knowledge ............... 7
  Unit 1: (i) Sets and Functions (ii) Composite Functions  7
  Unit 2: Binary Operations .......................... 7
  Unit 3: Trigonometry ............................... 7

Time ....................................................... 7
Material ............................................... 7
Module Rationale ..................................... 8

CONTENT ............................................. 8

Unit 1: (i) Sets and Functions (ii) Composite Functions 8
Unit 2: Binary Operations .......................... 8
Unit 3: Trigonometry ............................... 9

Outline ................................................ 9

General Objective(s) .................................. 10
  Specific Learning Objectives (Instructional Objectives) 10

TEACHING AND LEARNING ACTIVITIES ........... 11

Module 1: Basic Mathematics, Pre-assessment ...... 11
  Unit 1: Sets and Functions ........................ 11

Unit 1: Pre-assessment Solutions .................. 12
  Unit 2: Binary Operations ........................ 12

Unit 2: Pre-assessment Solutions .................. 14
  Unit 3: Trigonometry ............................... 14

Unit 3: Pre-assessment Solutions .................. 15
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COMPULSORY READINGS</strong></td>
<td>17</td>
</tr>
<tr>
<td>Reading #1</td>
<td>17</td>
</tr>
<tr>
<td>Reading #2</td>
<td>17</td>
</tr>
<tr>
<td><strong>Unit 1: Sets and Functions</strong></td>
<td>20</td>
</tr>
<tr>
<td>Module 1: Basic Mathematics</td>
<td>20</td>
</tr>
<tr>
<td>Unit 1, Activity 1: Sets and Functions</td>
<td>20</td>
</tr>
<tr>
<td>Learning Activities</td>
<td>20</td>
</tr>
<tr>
<td>Overview</td>
<td>20</td>
</tr>
<tr>
<td>Readings</td>
<td>21</td>
</tr>
<tr>
<td>Module 1: Basic Mathematics</td>
<td>26</td>
</tr>
<tr>
<td>Unit 1, Activity 2: Composite functions</td>
<td>26</td>
</tr>
<tr>
<td><strong>Unit 2: Binary Operations</strong></td>
<td>31</td>
</tr>
<tr>
<td>Specific Learning Objectives</td>
<td>31</td>
</tr>
<tr>
<td>Overview</td>
<td>31</td>
</tr>
<tr>
<td>Readings</td>
<td>31</td>
</tr>
<tr>
<td><strong>Unit 3: Trigonometry</strong></td>
<td>35</td>
</tr>
<tr>
<td>Specific Objectives</td>
<td>35</td>
</tr>
<tr>
<td>Overview</td>
<td>35</td>
</tr>
<tr>
<td>Unit 3, Activity 1: Angles and Laws of Triangle</td>
<td>37</td>
</tr>
<tr>
<td>Unit 3, Activity 2: Angle Measures and Trigonometric identities</td>
<td>43</td>
</tr>
<tr>
<td>Unit 3, Activity 3: Graphing Trigonometric functions</td>
<td>47</td>
</tr>
<tr>
<td><strong>Summative Evaluation</strong></td>
<td>52</td>
</tr>
<tr>
<td>Unit 1: Summative Assessment Solutions</td>
<td>54</td>
</tr>
<tr>
<td>Unit 2: Summative Assessment Questions</td>
<td>58</td>
</tr>
<tr>
<td>Unit 2: Summative Assessment Answers</td>
<td>59</td>
</tr>
<tr>
<td>Unit 3: Summative Assessment Questions</td>
<td>61</td>
</tr>
<tr>
<td>Unit 3: Summative Assessment Solutions</td>
<td>62</td>
</tr>
<tr>
<td><strong>References</strong></td>
<td>63</td>
</tr>
<tr>
<td>Main Author of the Module</td>
<td>63</td>
</tr>
</tbody>
</table>
Introduction

Title of Module
Mathematics 1, Basic Mathematics, Prof. Jairus. Khalagai, University of Nairobi.

Prerequisite Courses or Knowledge

Unit 1: (i) Sets and Functions (ii) Composite Functions
Secondary school mathematics is prerequisite.
This is a level 1 course.

Unit 2: Binary Operations
Basic Mathematics 1 is prerequisite.
This is a level 1 course.

Unit 3: Trigonometry
Basic Mathematics 2 is prerequisite.
This is a level 2 course.

Time
120 hours

Material
The course materials for this module consist of:

- Study materials (print, CD, on-line)
- (pre-assessment materials contained within the study materials)
- Two formative assessment activities per unit (always available but with specified submission date). (CD, on-line)
- References and Readings from open-source sources (CD, on-line)
- ICT Activity files
• Those which rely on copyright software
• Those which rely on open source software
• Those which stand alone
• Video files
• Audio files (with tape version)
• Open source software installation files
• Graphical calculators and licenced software where available
• (Note: exact details to be specified when activities completed)

Module Rationale

The rationale of teaching Basic mathematics is that it plays the role of filling up gaps that the student teacher could be having from secondary school mathematics. For instance, a lack of a proper grasp of the real number system and elementary functions etc. It also serves as the launching pad to University Mathematics by introducing the learner to the science of reasoning called logic and other related topics.

CONTENT

Overview

This module consists of three units which are as follows:

Unit 1: (i) Sets and Functions (ii) Composite Functions

This unit starts with the concept of a set. It then introduces logic which gives the learner techniques for distinguishing between correct and incorrect arguments using propositions and their connectives. A grasp of sets of real numbers on which we define elementary functions is essential. The need to have pictorial representations of a function necessitates the study of its graph. Note that the concept of a function can also be viewed as an instruction to be carried out on a set of objects. This necessitates the study of arrangements of objects in a certain order, called permutations and combinations.

Unit 2: Binary Operations

In this unit we look at the concept of binary operations. This leads to the study of elementary properties of integers such as congruence. The introduction to algebraic structures is simply what we require to pave the way for unit 3.
Unit 3: Trigonometry

In this unit, students will learn the basic techniques of angles measurement in triangles. The triangles laws and the trigonometric identities will be presented. The graphing of trigonometric functions will be discussed.

Outline

Unit 1: (i) Sets and Functions (ii) Composite Functions (50 hours)

Level 1. Priority A. No prerequisite.

Sets (4 hours)

Elementary logic (8 hours)

Number systems (6 hours)

Complex numbers (4 hours)

Relations and functions (8 hours)

Elementary functions and their graphs (8 hours)

Permutations (7 hours)

Combinations (5 hours)

Unit 2: Binary Operations (35 hours)

Level 1. Priority A. Basic Mathematics 1 is prerequisite.

Binary operations. (7 hours)

Elementary properties of integers. (7 hours)

Congruence. (7 hours)

Introduction to Algebraic structures. (7 hours)

Applications (7 hours)

Unit 3: Trigonometry (35 hours)

Level 2. Priority B. Basic Mathematics 2 is prerequisite.

Conversion angles from Degrees to Radians

Triangles Laws (2 hours)

Trigonometric functions and graphing (7 hours)
Trigonometric Identities. (5 hours)

This diagram shows how the different sections of this module relate to each other. The central or core concept is in the centre of the diagram. (Shown in red).

Concepts that depend on each other are shown by a line.

For example: Set is the central concept. The Real Number System depends on the idea of a set. The Complex Number System depend on the Real Number System.

**General Objective(s)**

You will be equipped with knowledge of elementary mathematical logic, sets, numbers and algebraic structures required for effective teaching of mathematics in secondary schools.

**Specific Learning Objectives (Instructional Objectives)**

By the end of this module, the learner should be able to…”

construct mathematical arguments.

make connections and communicate mathematical ideas effectively and economically.

identify patterns, make abstractions and generalize.

analyze various mathematical structures and the similarities and differences among these structures.

identify and calculate angles

construct using the correct measurement state the trigonometric laws and identities
Teaching And Learning Activities

Module 1: Basic Mathematics, Pre-assessment

Unit 1: Sets and Functions

Assessments and Solutions

Pre-assessment Questions

Given the quadratic equation:

\[ 2x^2 - x - 6 = 0 \]

The roots are:

\[ \{-4, 3\} \]
\[ \{4, -3\} \]
\[ \left\{ \frac{2}{3}, \frac{1}{2} \right\} \]
\[ \left\{ -\frac{2}{3}, \frac{3}{2} \right\} \]

The value of the function \( f(x) = 2x^2 + 3x + 1 \) at \( x = 3 \) is:

19
28
46
16

Which of the following diagrams below represents the graph of \( y = 3x(2 - x) \)

The solution of the equation

\[ \sin x = -\frac{1}{2} \text{ in the range } 0 \leq x^* \leq 360 \text{ is:} \]
Given the triangle $ABC$ below

Which of the following statements is correct?

$$\cos \alpha = \frac{2}{\sqrt{15}}$$

$$\sin \alpha = \frac{\sqrt{5}}{2}$$

$$\tan \alpha = 2$$

$$\sec \alpha = \frac{1}{\sqrt{5}}$$

**Unit 1: Pre-assessment Solutions**

The following are the answers to the multiple choice questions.

Q 1 c  Q 2 b  Q 3 b  Q 4 c  Q 5 c

**Unit 2: Binary Operations**

1. The inverse of the function

$$f(x) = \frac{1}{x - 1}$$

is

- (a) $f^{-1}(x) = x - 1$
- (b) $f^{-1}(x) = \frac{1 - x}{x}$
2. If \( \sin \frac{x}{2} = \frac{a}{2} \) then

\[ \sin x \text{ in terms of } a \text{ is:} \]

(a) \( \frac{a}{\sqrt{4-a^2}} \)

(b) \( a\sqrt{4-a^2} \)

(c) \( a \)

(d) \( \frac{\sqrt{4-a^2}}{2} \)

3. A girl has 3 skirts, 5 blouses and 4 scarves. The number of different outfits consisting of skirt, blouse and scarf that she can make out of these is:

220

60

12

150

4. Given the complex number

\( z = 1 - i \) we have that \( \text{Arg } z \) is:

450

1350

2250
If \( a \times b = a^2 + ab - 1 \), then

\[ 5 \times 3 = \] 15

3150

Unit 2: Pre-assessment Solutions

Q1. c  Q2. b  Q3. b  Q4. b  Q5. a

Unit 3: Trigonometry

1. If the hypotenuse of a right triangle is of length 5 and one leg is of length 2, what is the length of the other leg?

\[ 2\sqrt{3} \]

\[ \sqrt{21} \]

\[ \sqrt{7} \]

2. If \( \frac{3}{5} \) find \( \frac{3}{5} \).

\[ 2/5 \]

\[ 5/4 \]

\[ 4/5 \]

\[ 1/3 \]

3. Express \( \sin 57 \cos w + \cos 57 \sin w \) as a single term

\( \sin (57+w) \)
cos(57+w)

sin(57-w)

cos(57-w)

4. What is the measure of angle A in the right triangle below?

170
370
270
900

5. In a right triangle, the measure of one of the angles is 49° and the hypotenuse has a length of 50 cm. Which of the following is the nearest approximation to the length, in cm, of the leg opposite to this angle?

32.8
57.5
30.3
37.7

**Unit 3: Pre-assessment Solutions**

1. b  
2. c  
3. a  
4. c  
5. d

Title of Pre-assessment: _____________________________

PEDAGOGICAL COMMENT FOR LEARNERS

(100-200 words)

The questions in this pre-assessment are designed to test your readiness for studying the module.
The 5 questions preparing you for unit 1 require high school mathematics. If you make any errors, this should suggest the need to re-visit the high school mathematical topic referred to in the question.

The questions for unit 2 and unit 3 test your readiness after having completed the learning activities for unit 1 and unit 2.

If you make errors in the unit 2 pre-assessment, you should check through your work on unit 1 in this module. Likewise, If you make errors in the unit 3 pre-assessment, you should check through your work on unit 2 in this module.

**KEY CONCEPTS (GLOSSARY)**

**Algebraic structure:** This is the collection of a given set G together with a binary operation * that satisfies a given set of axioms.

**Binary operation:** This is a mapping which assigns to each ordered pair of elements of a set G, exactly one element of G.

**Composite Function:** This is a function obtained by combing two or more other simple functions in a given order.

**Function:** This is a special type of mapping where an object is mapped to a unique image.

**Mapping:** This is simply a relationship between any two given sets.

**Proposition:** This is a statement with truth value. Thus we can tell whether it is true or false

**Set:** This is a collection of objects or items with same properties

**Trigonometry:** The measurement of a triangle

**Radian:** A radian is the measure of the angle that cuts off an arc of length 1 on the unit circle.

**Angle:** An angle is when two rays (think of a ray as “half” of a line) have their end point in common.

**Angle of elevation:** The angle measure from the horizon or horizontal line, up.
COMPULSORY READINGS

Reading #1
A Textbook for High School Students Studying Maths by the Free High School Science Texts authors, 2005, pg 38-47 (File name on CD: Secondary_School_Maths)

Reading #2
Sets relations and functions by Ivo Duntsch and Gunther Gediga methodos publishers (UK) 2000. (File name on CD: Sets_Relations_Functions_Duntsch)

General Abstract and Rationale
All of the compulsory readings are complete open source textbooks. Together they provide more than enough material to support the course. However, the text contains specific page references to activities, readings and exercises which are referenced in the learning activities.

COMPULSORY RESOURCES
If you want to visit any of the website listed, copy and paste the links to a new browser

Wolfram MathWorld (visited 23.03.16)
http://mathworld.wolfram.com/

A complete and comprehensive guide to all topics in mathematics. The students is expected to become familiar with this web site and to follow up key words and module topics at the site. http://www.wikipedia.org/.

Wikipedia provides encyclopedic coverage of all mathematical topics. Students should follow up key words by searching at wikipedia.

USEFUL LINKS
Set Theory (visited 23.03.16)
http://www.mathresource.iitb.ac.in/project/indexproject.html

Read through any of the sections by clicking on the slices of the pie.

Especially work through the section called ‘functions’.

Click the NEXT link at the bottom of the page to move forward.

Click on double arrow buttons to see things move!
Teaching And Learning Activities

Wolfram MathWorld (visited 23.03.16)

http://mathworld.wolfram.com/SetTheory.html

Read this entry for Set Theory.

Follow links to explain specific concepts as you need to.

Wikipedia

http://www.wikipedia.org/

Type ‘Set Theory’ into the search box and press ENTER.

Follow links to explain specific concepts as you need to.

MacTutor History of Mathematics

http://www-history.mcs.st-andrews.ac.uk/HistTopics/Beginnings_of_set_theory.html

Read for interest the history of Set Theory

Composite Functions

http://www.bbc.co.uk/education/aspuru/maths/13pure/02functions/06composite/index.shtml

Read through the first page

Use the arrow buttons at the bottom of the page to move to the next page

Page 2 is an interactive activity. Work through it carefully.

Read page 3 for details on notation.

Test your understanding on page 4.

Wolfram MathWorld

http://mathworld.wolfram.com/Composition.html

Read this entry for Composite Functions.

Follow links to explain specific concepts as you need to.

Wikipedia (visited 23.03.16)

https://www.wikipedia.org/

Type ‘Composite Functions’ into the search box and press ENTER.
Follow links to explain specific concepts as you need to.

**Wolfram MathWorld (visited 23.03.16)**

http://mathworld.wolfram.com/BinaryOperation.html

Read this entry for Binary Operations.

Copy and paste and follow links to explain specific concepts as you need to.

**Wikipedia (visited 23.03.16)**

http://www.wikipedia.org/

Type ‘Binary Operations’ into the search box and press ENTER.

Follow links to explain specific concepts as you need to.

**GeoGebra (visited 23.03.16)**

https://www.geogebra.org/download

Use geogebra in plotting graph. If you don’t have have it you can download it from the link above.

**MIT Website (visited 23.03.16)**

http://web.mit.edu/jorloff/www/18.01a-esg/OCWTrig.pdf
Learning Activities

Specific Learning Objectives

By the end of this activity, the learner should be able to:

- distinguish between a function and a general mapping
- demonstrate relationship between sets and functions
- state examples of sets of real numbers and some functions defined on such sets

Overview

The notions of a set and a function are the most fundamental concepts which together constitute the foundations of Mathematics. Indeed, different branches of Mathematics start with these two fundamental concepts.

In this activity, we are simply demonstrating how sets of objects are easily extracted from our surroundings. In particular, we are going to motivate the learner to be able to easily come up with examples of general mapping and functions defined on sets of real numbers.

We note that it is of great importance for the learner to be able to distinguish between a general mapping and a function diagrammatically. This will help the learner in grasping many properties about functions in higher courses.

KEY CONCEPTS

**Function:** This is a special type of mapping where an object is mapped to a unique image.

**Mapping:** This is simply a relationship between any two given sets

**Proposition:** This is a statement with truth value. Thus we can tell whether it is true or false

**Set:** This is a collection of objects or items with same properties
Readings

All of the readings for the module come from Open Source text books. This means that the authors have made them available for any reader to use them without charge. We have provided complete copies of these texts on the CD accompanying this course.

A Textbook for High School Students Studying Maths by the Free High School Science Texts authors, 2005, pg 38-47 (File name on CD: Secondary_School_Maths)

Elements of Abstract and Linear Algebra by E. H. Connell, 1999, University of Miami, pg. 1-13 (File name on CD: Abstract_and_linear_algebra_Connell)

Internet Resources

Set Theory (visited 23.03.16)

http://www.mathresource.iitb.ac.in/project/indexproject.html

Read through any of the sections by clicking on the slices of the pie.

Especially work through the section called ‘functions’.

Click the NEXT link at the bottom of the page to move forward.

Click on double arrow buttons to see things move!

Wolfram MathWorld (visited 23.03.16)

http://mathworld.wolfram.com/SetTheory.html

Read this entry for Set Theory.

Follow links to explain specific concepts as you need to.

Wikipedia (visited 23.03.16)

http://www.wikipedia.org/

Type ‘Set Theory’ into the search box and press ENTER.

Follow links to explain specific concepts as you need to.

MacTutor History of Mathematics (visited 23.03.16)

http://www-history.mcs.st-andrews.ac.uk/HistTopics/Beginnings_of_set_theory.html

Read for interest the history of Set Theory
Introduction

Story of Maize Grinding Machine

Jane walks in a village to a nearby market carrying a basket of maize to be ground into flour. She puts the maize into a container in the grinding machine and starts rotating the handle. The maize is then ground into flour which comes out of the machine for her to take home.

Question

What relation can you make among the maize, the grinding machine and the flour?

Story of children born on the Christmas day in the year 2005
It was reported on the 25th of December 2005 in Pumwani Maternity Hospital which is in Nairobi the Capital City of Kenya that mothers who gave birth to single babies were a total of 52. This was the highest tally on that occasion. As it is always the case each baby was given a tag to identify him or her with the mother.

Questions
In the situation above given the mother how do we trace the baby?
Given the baby how do we trace the mother?

Activity
Note that we can now represent the story of the maize grinding machine diagrammatically as follows:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Set of some content (in this case maize) to be put in the grinding machine</td>
</tr>
<tr>
<td>f</td>
<td>The mapping or function representing the process in the grinding machine</td>
</tr>
<tr>
<td>B</td>
<td>Set of the product content (in this case flour) to be obtained</td>
</tr>
</tbody>
</table>
**Example**

In this example we define two sets and a relation between them as follows:

Let \( A = \{2, 3, 4\} \)

\( B = \{2, 4, 6, 8\} \)

\( f \) is a relationship which says “is a factor of”

e.g. 3 is a factor of 6

In this case we have the following mapping:

**Example**

Think of a number of such situations and represent them with a mapping diagram as shown above.

In our second story of each mother giving birth to only one child can be represented in a mapping diagram as follows:

\[
\begin{array}{cc}
A & B \\
\end{array}
\]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set of babies</td>
<td>Set of mothers</td>
</tr>
<tr>
<td>Set of mothers</td>
<td>Relation which says “baby to”</td>
</tr>
</tbody>
</table>

**Remarks**

Notice that in this mapping each object is mapped onto a unique image. In this case it is a function. We write \( f: A \rightarrow B \)

Note also that in the mapping above even if we interchanged the roles of sets \( A \) and \( B \) we still have that each object has a unique image. Thus we have…

\[
\begin{array}{cc}
B & A \\
\end{array}
\]

In this case we have

<table>
<thead>
<tr>
<th>B</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set of mothers</td>
<td>Set of babies</td>
</tr>
<tr>
<td>Relation which says “is mother of”</td>
<td></td>
</tr>
</tbody>
</table>

In this case we say that the function \( f \) has an inverse \( g \). We normally denote this inverse \( g \) as \( f^{-1} \)
Thus for f: A → B we have f⁻¹: B → A

Example 1.1.6.4

Let A = \{1, 2, 3, 4, 5\}
B = \{2, 3, 5, 7, 9, 11, 12\}
f : x ↦ 2x + 1

Then we have the mapping as follows: f(x) → 2x + 1

For notation purposes in this mapping we can also write:
f(1) = 3, f(2) = 5 etc
In general f(x) = 2x + 1

The set A is called the domain of f and the set B is called the co-domain of f.
The set \{3, 5, 7, 9, 11\} within 13 on which all elements of A are mapped is called range of f.
Note that here the inverse of f is given by \( f^{-1}(x) = \frac{x - 1}{2} \) and is also a function.

Exercise

Starting with the set
A = \{2, 4, 7, 9, 11, 12\} as the domain find the range for each of the following functions.
f(x) = 3x – 2
g(x) = 2x² + 1
h(x) = \( \frac{x}{1 - x} \)

Exercise

State the inverse of the following functions:
a) \( f(x) = 3 - \frac{2}{x} \)
b) \( g(x) = \frac{1}{1 - x} \)
c) \( h(x) = 3x^2 - 2 \)
Exercise

Using as many different sets of real numbers as domains give examples of the following:

- A mapping which is not a function
- A mapping which is a function
- A function whose inverse is not a function
- A function whose inverse is also a function

Demonstrate each example on a mapping diagram. If you are in a group each member should come up with an example of his or her own for each of the cases above.

Module 1: Basic Mathematics

Unit 1, Activity 2: Composite functions

Specific Objectives

By the end of this activity, the learner should be able to:

demonstrate a situation in which two consecutive instructions issued in two different orders may yield different results.

verify that two elementary functions operated (one after another) in two different orders may yield different composite functions.

draw and examine graphs of different classes of functions starting with linear, quadratic etc.

Overview

Composite functions are about combinations of different simple mappings in order to yield one function. The process of combining even two simple statements in real life situations in order to yield one compound statement is important. Indeed the order in which two consecutive instruction are issued must be seriously considered so that we do not end up with some embarrassing results.

In this activity we are set to verify that two elementary functions whose formulae are known if combined in a certain order will yield one composite formula and if order in which they are combined is reversed then this may yield a different formula.

We note here that it is equally important to be able to represent a composite function pictorially by drawing its graph and examine the shape. Indeed, the learner will be able to draw these graphs starting with linear functions quadratic and even trigonometric functions etc.
**KEY CONCEPTS**

**Composite Function:** This is a function obtained by combing two or more other simple functions in a given order.

---

**Readings**

All of the readings for the module come from Open Source text books. This means that the authors have made them available for any reader to use them without charge. We have provided complete copies of these texts on the CD accompanying this course.

Sets relations and functions by Ivo Duntsch and Gunther Gediga, Methodos publishers (UK) 2000. (File name on CD: Sets_Relations_Functions_Duntsch)

**Internet Resources**

Composite Functions (visited 23.03.16)

http://www.bbc.co.uk/education/guides/zc6hhyc/revision/4

**Read through the first page**

Use the arrow buttons at the bottom of the page to move to the next page

Page 2 is an interactive activity. Work through it carefully.

Read page 3 for details on notation.

Test your understanding on page 4.

Wolfram MathWorld (visited 23.03.16)

http://mathworld.wolfram.com/Composition.html

Read this entry for Composite Functions.

Follow links to explain specific concepts as you need to.

Wikipedia (visited 23.03.16)

http://www.wikipedia.org/

Type ‘Composite Functions’ into the search box and press ENTER.

Follow links to explain specific concepts as you need to.
Introduction:

A Story of Nursery School Children

Two Children brother and sister called John and Jane go to a Nursery school called Little Friends.

One morning they woke up late and found themselves in a hurry to put on clothes and run to school, Jane first put on socks then shoes. But her brother John first put on shoes then socks. Jane looked at him and burst into laughter as she run to school to be followed by her brother.

Question

Why did Jane burst into laughter?

Story of a visit to a beer brewing factory

A science Club in a secondary school called Nabumali High School in Uganda, one Saturday made a trip to Jinja town to observe different stages of brewing beer called Nile Beer. It was noted that of special interest was the way some equipment used in the process would enter some chamber and emerge transformed. For example an empty bottle would enter a chamber and emerge transformed full of Nile Beer but without the bottle top. Then it would enter the next chamber and emerge with the bottle top on.
**Question**

Can you try to explain what happens in each chamber of the brewing factory?

**Activity**

We note that in our story of the Nursery school Children what is at stake is the order in which we should take instruction in real life situations. Jane laughed at her brother because she saw the socks on top of the shoes. In other words her brother had ended up with composite instruction or function which was untenable. We can also look at other such cases through the following example.

**Example**

I think of a number, square it then add 3 or I think of a number add 3 then square it. If we let the number to be $x$, then we will end up with two different results namely $x^2 + 3$ and $(x + 3)^2$ respectively.

**Example**

Can you now come up with a number of examples similar to the one above?

If we now consider our story on the brewing of Uganda Warangi we note that each Chamber has a specific instruction on the job to perform. This is why whatever item passes through the chamber must emerge transformed in some way. We can also look at an example where instructions are given in functional form with explicit formulae as shown below.

**Example**

Consider the composition of the functions.

$$f : x \rightarrow 2x \quad \text{and} \quad g : x \rightarrow x + 5$$

Here if we are operating $f$ followed by $g$ then we double $x$ before we add 5. But if operate $g$ followed by $f$ then we add 5 to $x$ before we double the result.

**For notation purposes**

$$(f \circ g)(x) = f(g(x)) \text{ means } g \text{ then } f. \quad \text{While } (g \circ f)(x) = g(f(x)) \text{ means } f \text{ then } g$$

Representing the composite function $g(f(x)) = 2x + 5$
While:
representing the composite function \( f(g(x)) = 2(x + 5) \)

**Exercise**

Given \( f : x \rightarrow 3x + 1 \quad g : x \rightarrow x - 2 \)

Determine the following functions:

(a) \( f \circ g \)

(b) \( g \circ f \)

(c) \( (f \circ g)^{-1} \)

(d) \( (g \circ f)^{-1} \)

Taking \( x = 3 \) draw a diagram for each of the composite functions above as is the case in example 1.2.6.3 above.

**Exercise**

Sketch the graph for each of the following function: assuming the domain for each one of them is the whole set of real numbers.

\[
f(x) = 2x - 3 \\
g(x) = 4x^2 - 12x \\
h(x) = x^3 - 3x + 1 \\
k(x) = 2 \sin x
\]
Unit 2: Binary Operations

Specific Learning Objectives

By the end of this activity, the learner should be able to:

- Give examples of binary operations on various operations
- Determine properties of commutativity or associativity on some binary operations.
- Determine some equivalence relations on some algebraic structures

Overview

The concept of a binary operation is essential in the sense that it leads to the creation of algebraic structures.

The well known binary operations like + (addition) and \( \times \) (multiplication) do constitute the set of real numbers as one of the most familiar algebraic structures. Indeed the properties of commutativity or associativity can easily be verified with respect to these operations on \( \mathbb{R} \).

However, in this activity we define and deal with more general binary operations which are usually denoted by \( * \).

For example for any pair of points \( x \) and \( y \), in a given set say \( G \), \( x * y \) could even mean pick the larger of the two points. It is clear here that \( x * y = y * x \). Consequently we will exhibit examples of more general algebraic structures that arise from such binary operations.

### KEY CONCEPTS

**Algebraic structure:** This is the collection of a given set \( G \) together with a binary operation \( * \) that satisfies a given set of axioms.

**Binary operation:** This is a mapping which assigns to each ordered pair of elements of a set \( G \), exactly one element of \( G \).

Readings

All of the readings for the module come from Open Source text books. This means that the authors have made them available for any reader to use them without charge. We have provided complete copies of these texts on the CD accompanying this course.

Sets of relations and functions by Ivo Duntsch and Gunther Gediga, Methodos Publishers, UK, 2000 pp 30-34 (File name on CD: Sets_Relations_Functions_Duntsch)
Internet Resources

Binary Color Device (visited 23.03.16)


This is a puzzle involving binary operations and group tables. Use the puzzle to develop your understanding.

Wolfram MathWorld (visited 23.03.16)

http://mathworld.wolfram.com/BinaryOperation.html

Read this entry for Binary Operations.

Follow links to explain specific concepts as you need to.

Wikipedia (visited 23.03.16)

http://www.wikipedia.org/

Type ‘Binary Operations’ into the search box and press ENTER.

Follow links to explain specific concepts as you need to.

Introduction: The Story of the Reproductive System

In a real life situation among human beings, you will find that an individual gets into a relation with another individual of opposite sex. They then reproduce other individuals who constitute a family. We then have that families with a common relationship will constitute a clan and different clans will give rise to a tribe etc…

We note that even in ecology the same story can be told. For example we can start with an individual like an organism which is able to reproduce other organisms of the same species that will later constitute a population. If different populations stay together then they will constitute a community etc…
**Question:**

What is the mechanism that can bring together two individuals (human beings or organisms of ecology) in order to start reproduction?

**Activity**

We note that in the case of human beings in our story above we could say that it is marriage that brings together a man and woman to later constitute a family after reproduction. In mathematics the concept of marriage could be looked at as a binary operation between the two individuals. If we can reflect on our mapping diagram we have the following:

Where $A =$ set of men wedding in a given time

$B =$ set of women getting marriage at the same time

$* =$ operation which says $x$ weds $y$

Clearly $x * y = y * x$

In this case this particular binary operation is commutative. If we denote the relation implied by the binary operation $*$ by $R$ then we write, $xRy$ to mean $x$ is related to $y$ or $yRx$ to mean $y$ is related to $x$. If $xRy$ $yRx$ then the relation is said to be symmetric.

**Question**

Can you try to define some relations on sets of your choice and check whether they are symmetric?

In general we note that if a binary operation $*$ gives rise to a relation $R$ then:

- $R$ is reflexive if $xRx$
- $R$ is symmetric if $xRy$ $yRx$
- $R$ is transitive if $xRy$ and $yRz$ $xRz$
- For all elements $x,y,z$ in a given set

We also note that a relation $R$ satisfying all the three properties of reflexive, symmetric and transitive above is said to be an equivalence relation.

**Example**

Let $U$ be the set of all people in a community.

Which of the following is an equivalence relation among them?

is an uncle of
is a brother of

We note that in part (i) if R is the relation “is an uncle of” then xRy does not imply yRx. Thus R is not symmetric in particular. Hence R is not an equivalence relation.

However in part (ii) if R is the relation “is a brother of” then xRx is valid.

Also xRy  yRx and finally xRy and yRz  xRz.

Hence R is an equivalence relation.

**Exercise**

Which of the following is an equivalence relation on the set of all human beings?

- is a friend of
- is a relative of

**Exercise**

Determine whether the binary operation * on the set of real numbers is commutative or associative in each of the following cases

- $x * y = y2x$
- $x * y = xy + x$

Define a relation ~ on the set of integers as follows

$\text{a } \sim \text{ b if and only if a + b is even.}$

Determine whether ~ is an equivalence relation on .

Give an example of an equivalence relation on the set of real numbers.

If you are working in a group each member of the group should give one such example.

Complete exercise 2.4.1 p 34 in Sets, Relations and Functions by Duntsch and Gediga (solutions on pp. 48 – 49)
Unit 3: Trigonometry

Specific Objectives

By the end of this activity, the learner should be able to:

- convert degrees into radian Measures
- define and identify the different types of angles
- state theorems and axioms in trigonometry
- state the trigonometric identities and use them to solve trigonometric equations
- Draw a Graph the common trigonometric functions
- prove the theorems and give examples

Overview

As you probably know, trigonometry is just “the measurement of triangles”, and that is how it got started, in connection with surveying the earth and the universe. But it has become an essential part of the language of mathematics, physics, and engineering. Hence, in this unit students will have the opportunity to understand the importance of angles and how to evaluate using either degree or radian.

KEY CONCEPTS

Radian: A radian is the measure of the angle that cuts off an arc of length 1 on the unit circle.

Angle: An angle is when two rays (think of a ray as “half” of a line) have their end point in common.

Angle of elevation: The angle measure from the horizon or horizontal line, up.
**Angle of Depression:** The angle measured from the horizon or horizontal line, down.

**Supplementary Angle:** Two angles are supplementary if their sum equals to 180°.

**Complimentary Angle:** Two angles are complementary if their sum equals to 90°.

**Congruent Angle:** Two angles are congruent if they are equal.

**Isosceles triangle:** Triangle that have two sides of equal length.

**Equilateral triangle:** All the three sides have equal length.

**Right-Angle triangle:** The two sides of the triangle are perpendicular.

**Pythagorean Theorem:** Given a right angle triangle with side a, b and c (c being the longest side called hypotenuse), then \(a^2 + b^2 = c^2\).

**Law of Sine:** If a triangle have sides of length a, b and c opposite the angles A, B and C respectively.

**Law of Cosine:** If a triangle have sides of length a, b and c opposite the angles A, B and C respectively.

**Readings**

All of the readings for the module come from Open Source text books. This means that the authors have made them available for any reader to use them without charge. We have provided complete copies of these texts on the CD accompanying this course.

**Internet Resources**


http://www.ck12.org/geometry/Trigonometry-Word-Problems/lesson/
Introduction: Building a wheelchair ramp

What if a restaurant needed to build a wheelchair ramp for its customers? The angle of elevation for a ramp is recommended to be 5°. If the vertical distance from the sidewalk to the front door is two feet, what is the horizontal distance that the ramp will take up (x)? How long will the ramp be (y)? Round your answers to the nearest hundredth. After completing this Concept, you’ll be able to use trigonometry to solve this problem.

Answers:

To find the horizontal length and the actual length of the ramp, we need to use the tangent and sine.

\[ \tan 5° = \frac{2}{x} \]

and \[ \sin 5° = \frac{2}{y} \]

\[ x = \frac{2}{\sin 5°} \approx 22.86 \]

and \[ y = \frac{2}{\tan 5°} = \frac{22.95}{5} = 22.95 \]

Module 1: Basic Mathematics

Unit 3, Activity 1: Angles and Laws of Triangle

Learning Activities
Specific Learning Objectives

By the end of this activity, the learner should be able to:

- identify the type of angles
- state the Pythagorean theorem
- perform calculation of length of triangles
- find the angles of right triangle
- calculate the angle of elevation and depression

Overview

Since trigonometry is important in day to day activities, understanding the basic concepts is of interest to students. This activity will introduce students to the different types of triangles and laws related to them. The Pythagorean theorem will be discussed and its importance.

**KEY CONCEPTS**

**Radian:** A radian is the measure of the angle that cuts off an arc of length 1 on the unit circle.

**Angle:** An angle is the rotation required to superimpose one of two intersecting lines on the other.

A right angle is an angle with measure equal to 90 degrees.
An acute angle is an angle with a measure between 0 and 90 degrees.
An obtuse angle is an angle with a measure between 90 and 180 degrees.

**Angle of elevation:** The angle measure from the horizon or horizontal line, up.

**Angle of Depression:** The angle measured from the horizon or horizontal line, down.

**Supplementary Angle:** Two angles are supplementary if the sum of their measures equals to 180°

**Complimentary Angle:** Two angles are complementary if the sum of their measures equals to 90°

**Congruent Angle:** Two angles are congruent if their measures are equal.

Readings

Introduction

Since Trigonometry literally means measuring a figure which is having three sides, his activity will help the students to understand how to measure angles and perform the basic calculation related to it. It also enable them to understand theorems of triangles. And angles can be represented in two ways: Degree or radian.

Activity

Note that there are different types of triangles and laws characterizing them. To know the values of angles in a triangles, students must used the laws to compute them. As in the introduction, in building ramp, houses, a certain type of inclination is needed for the roof so that water can run over it. This is an issue related to angle measurement.

To calculate the angle of elevation or depression, the laws of triangles and the Pythagorean theorem are used.

Example

For each triangle below, determine the unknown angle(s):

For triangle \( \triangle ABC \), \( A = 35^\circ \) and \( C = 20^\circ \), and we know that \( A+B+C = 180^\circ \), so

\[ 35^\circ + B + 20^\circ = 180^\circ \Rightarrow B = 180^\circ - 35^\circ - 20^\circ \Rightarrow B = 125^\circ . \]

For the right triangle \( \triangle DEF \), \( E = 53^\circ \) and \( F = 90^\circ \), and we know that the two acute angles D and E are complementary, so

\[ D + E = 90^\circ \Rightarrow D = 90^\circ - 53^\circ \Rightarrow D = 37^\circ . \]
Unit 3: Trigonometry

For triangle $\triangle XYZ$, the angles are in terms of an unknown number $\alpha$, but we do know that $X + Y + Z = 180^\circ$, which we can use to solve for $\alpha$ and then use that to solve for $X$, $Y$, and $Z$:

\[\alpha + 3\alpha + \alpha = 180^\circ \Rightarrow 5\alpha = 180^\circ \Rightarrow \alpha = 36^\circ \Rightarrow X = 36^\circ, \ Y = 3 \times 36^\circ = 108^\circ, \ Z = 36^\circ\]

**Example**

A 17 ft ladder leaning against a wall has its foot 8 ft from the base of the wall. At what height is the top of the ladder touching the wall?

**Solution:** Let $h$ be the height at which the ladder touches the wall. We can assume that the ground makes a right angle with the wall, as in the picture on the right. Then we see that the ladder, ground, and wall form a right triangle with a hypotenuse of length 17 ft (the length of the ladder) and legs with lengths 8 ft and $h$ ft. So by the Pythagorean Theorem, we have

\[h^2 + 8^2 = 17^2 \Rightarrow h^2 = 289 - 64 = 225 \Rightarrow h = 15 \text{ ft} .\]

**Example**

For each right triangle below, determine the length of the unknown side:

**Solution:** For triangle $\triangle ABC$, the Pythagorean Theorem says that

\[a^2 + 4^2 = 5^2 \Rightarrow a^2 = 25 - 16 = 9 \Rightarrow a = 3 .\]

For triangle $\triangle DEF$, the Pythagorean Theorem says that

\[e^2 + 1^2 = 2^2 \Rightarrow e^2 = 4 - 1 = 3 \Rightarrow e = \sqrt{3} \]

For triangle $\triangle XYZ$, the Pythagorean Theorem says that

\[1^2 + 1^2 = z^2 \Rightarrow z^2 = 2 \Rightarrow z = \sqrt{2} \sqrt{2} = \sqrt{4} = 2\]
Example

How tall is the building?

Solution:

Example

How far up with the wall will the ladder reach?
solution:

There are two ways to solve this problem:

1. Pythagorean Theorem
2. Trigonometry

If the angle is given, use trigonometry.

- $\sin 59^\circ = \frac{x}{12}$
- $12 \cdot \sin 59^\circ = \frac{x \cdot 12}{12}$
- $10.286 \text{ ft} = x$
- $x \approx 10.3 \text{ ft (round to nearest tenth)}$

Exercises:

1. Given angles $a$, $b$, $c$, $d$, $e$ and $f$ with measures
   
   $a = 21^\circ$
   $b = 90.1^\circ$
   $c = 90^\circ$
   $d = 134.2^\circ$
   $e = 69^\circ$
   $f = 45.8^\circ$

   Which of these angles are acute?

   Which of these angles are obtuse?

   Which pairs of angles are complementary?

   Which pairs of angles are supplementary?

2. Find $x$ and $H$ in the right triangle below.

3. In a right triangle $ABC$, $\tan(A) = 3/4$. Find $\sin(A)$ and $\cos(A)$.

4. From the top of a 200 meters high building, the angle of depression to the bottom of a second building is 20 degrees. From the same point, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.
Module 1: Basic Mathematics

Unit 3, Activity 2: Angle Measures and Trigonometric identities

Learning Activities

Specific Learning Objectives

By the end of this activity, the learner should be able to:

- convert angle measure in degree to radian
- identify the trigonometric identities
- use the identities to solve problems

Overview

Angles are measured in two ways: Degrees and radians. Therefore it is of great importance to know the relationship between those two mode of measurement.

The trigonometric identities play a central role in the understanding of the basic concepts of geometry.

Key concepts

radian: An angle of 1 radian is defined to be the angle subtended by an arc of length 1 on a circle of radius 1.

Trigonometric identities:

1. \( \csc \theta = \frac{1}{\sin \theta} \) when \( \sin \theta \neq 0 \)

2. \( \sec \theta = \frac{1}{\cos \theta} \) when \( \cos \theta \neq 0 \)

3. \( \cot \theta = \frac{1}{\tan \theta} \) when \( \tan \theta \) is defined and not 0

4. \( \sin \theta = \frac{1}{\csc \theta} \) when \( \csc \theta \) is defined and not 0

5. \( \cos \theta = \frac{1}{\sec \theta} \) when \( \sec \theta \) is defined and not 0

6. \( \tan \theta = \frac{1}{\cot \theta} \) when \( \cot \theta \) is defined and not 0
You know that the various trig functions are closely related to one another. For example, \( \cos q = 1/ \sec q \). Since this is true for every \( q \), this is an identity, not an equation that can be solved for \( q \). Trigonometric identities are very useful for simplifying and manipulating many mathematical expressions. You ought to know a few of them.

For the purpose of converting the unit of measurement of angles, the following rule is used:

\[
\begin{align*}
\text{Degrees to radians:} & \quad x \text{ degrees} = \left( \frac{\pi}{180} \cdot x \right) \text{ radians} \\
\text{Radians to degrees:} & \quad x \text{ radians} = \left( \frac{180}{\pi} \cdot x \right) \text{ degrees}
\end{align*}
\]

Also the common identities are presented to the students:

Probably the most familiar, and also one of the most useful, is the one based on Pythagoras’ Theorem and the definitions of \( \sin \) and \( \cos \):

\[ c^2 = a^2 + b^2, \text{ with } \sin q = a/c, \cos q = b/c; \]

\[ (\sin q)^2 + (\cos q)^2 = 1 \]

Dividing through by \( \cos^2 q \) or \( \sin^2 q \) gives two other identities useful for calculus:

\[ \tan^2 q + 1 = \sec^2 q; \quad 1 + \cot^2 q = \csc^2 q. \]

Dividing through by \( \cos^2 q \) or \( \sin^2 q \) gives two other identities useful for calculus:

\[ \tan^2 q + 1 = \sec^2 q; \quad 1 + \cot^2 q = \csc^2 q. \]

Two other very simple identities are:
sin(-a) = - sin a; cos(-b) = cos b.

(We say that sin q is an odd function of q and cos q is an even function of q.)

Angle addition formulas: Given any two angles a and b,

\[
\sin(a + b) = \sin a \cos b + \cos a \sin b \\
\cos(a + b) = \cos a \cos b - \sin a \sin b
\]

If you learn these formulas, you can easily construct the formulas for sin or cos of the difference of two angles. (Just use the odd/even properties of sin and cos.)

**The Laws of Sine:**

For any triangle ABC, labeled as in the diagram:

![Diagram of a triangle](image)

This result follows from considering the length of a perpendicular drawn from any angle to the opposite side. Such a perpendicular (BN in our diagram) can be calculated in two ways:

BN = c sin A = a sin C.

either of the other two perpendiculars will complete the relationships.

We can use the law of sines to solve a triangle if we know one angle and the length of the side opposite to it, plus one other datum – either another angle or the length of another side. (We can always make use of the fact that the angles of any triangle add up to 180.)

**The Law of Cosines**

This is useful if we do not know the values of an angle and its opposite side. What that means, essentially, is that we are given the value of at most one angle. If this is the angle A in the standard diagram, the Law of Cosines states that:

\[
a^2 = b^2 + c^2 - 2bc \cos A.
\]

Thus, if b, c, and A are given, we can calculate the length of the third side, a.
Example

Convert 18° to radians

Solution: Using the conversion formula (4.2) for degrees to radians, we get
\[ 18° = \frac{18\pi}{180} \times 18 = \frac{18\pi}{10}. \]

Example

Convert \( \frac{\pi}{9} \) radians to degrees.

Solution: Using the conversion formula (4.3) for radians to degrees, we get
\[ \frac{\pi}{9} = \left( \frac{180\pi}{9} \right) = 20. \]

Example

Simplify \( \cos^2 \theta \cdot \tan^2 \theta \).

Solution: We can use formula to simplify:
\[ \cos^2 \theta \cdot \tan^2 \theta = \sin^2 \theta. \]

Example

Prove that \( \tan \theta + \cot \theta \).

\[ \cot \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta} \]

Solution:

We will expand the left side to show that it is equal to the right side.
\[ \tan \theta + \cot \theta \]

\[ \cot \theta = \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \]

Multiplying both by a factor of 1
\[ \tan \theta + \cot \theta \]
After getting a common denominator

\[
\tan \theta + \cot \theta = \frac{\tan^2 \theta + \cot^2 \theta}{\tan \theta \cot \theta} = \frac{\tan^2 \theta + \cot^2 \theta}{\frac{\tan \theta}{\cot \theta}}
\]

Exercise

In a triangle labeled as in the diagram above, let \( A = 30 \), \( a = 10 \), and \( C = 135 \). Find \( B \), \( b \), and \( c \).

2. In another triangle, suppose \( B = 50 \), \( b = 12 \), \( c = 15 \). Find \( A \), \( C \), and \( a \).

3. In a triangle labeled as above, \( a = 5 \), \( b = 10 \), and \( C = 135 \). Find \( c \), \( A \), and \( B \).

4. In another triangle, suppose \( a = 10 \), \( b = 20 \), \( c = 25 \). Find all the angles.

[Draw a reasonably good sketch to see what the triangle looks like, and remember that \( \sin \theta \) and \( \sin(180^\circ - \theta) \) are equal.]

Module 1: Basic Mathematics

Unit 3, Activity 3: Graphing Trigonometric functions

Learning Activities

Specific Learning Objectives

By the end of this activity, the learner should be able to:

- define the trigonometric function
- use a software (GeoGebra/ Gnuplot) and graph them
- use the graph to solve basic problems, find the period and amplitude of trigonometric functions

Overview

In this activity, students will learn how to plot the graph of a trigonometric function. But also, they will plot graph relation to the common function used in this activity. GeoGebra or Gnuplot will be used to graph the trigonometric functions discussed in the unit. Graphs will be used to find the period, amplitude and frequency of the function.
Internet Resources

Go to http://www.gnuplot.info/download.html

Gnuplot is a free, open-source software package for producing a variety of graphs. Versions are available for many operating systems. Gnuplot can be used to graph trigonometric functions.

Introduction

In this activity, students learn how to plot trigonometric graph manually. They are also introduced to the use of a software such as GeoGebra or Gnuplot when plotting graph.

Activity

The trigonometric functions can be graphed just like any other function, as we will see in this activity. In the graphs we will always use radians for the angle measure. The graph of the sine and cosine are explained in details. (refer to text)

5.1 Graphing the Trigonometric Functions

The first function we will graph is the sine function. We will describe a geometrical way to create the graph, using the unit circle. This is the circle of radius 1 in the xy-plane consisting of all points (x, y) which satisfy the equation $x^2 + y^2 = 1$.

We see in Figure 5.1.1 that any point on the unit circle has coordinates $(x, y) = (\cos \theta, \sin \theta)$, where $\theta$ is the angle that the line segment from the origin to $(x, y)$ makes with the positive x-axis (by definition of sine and cosine). So as the point $(x, y)$ goes around the circle, its y-coordinate is $\sin \theta$.

We thus get a correspondence between the y-coordinates of points on the unit circle and the values $f(\theta) = \sin \theta$, as shown by the horizontal lines from the unit circle to the graph of $f(\theta) = \sin \theta$ in Figure 5.1.2 for the angles $\theta = 0, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$.

we can extend the above picture to include angles from 0 to cas in the figure below
Since the trigonometric functions repeat every $2\pi$, the following graph of the function $f(\theta) = \sin \theta$ in the interval $[-2\pi, 2\pi]$:

![Graph of $f(\theta) = \sin \theta$](image)

To graph the cosine function, one can again use the unit circle area but there is an easier way. Since $\cos(\theta + 90^\circ) = \cos(\theta + 90^\circ)$ for all $\theta$, so $\cos(0) = \cos(0)$, $\cos(90^\circ) = \cos(90^\circ)$, $\cos(180^\circ) = \cos(180^\circ)$, and so on. In other words, the graph of the cosine function is just the graph of the sine function shifted to the left by $90^\circ = \frac{\pi}{2}$. as in the figure below.

![Graph of $y = \sin x$](image)
To graph the cosine function, we could again use the unit circle idea (using the $x$-coordinate of a point that moves around the circle), but there is an easier way. Recall from Section 5.1.4 that $\cos x = \sin (x + 90^\circ)$ for all $x$. So $\cos 0^\circ$ has the same value as $\sin 90^\circ$, $\cos 90^\circ$ has the same value as $\sin 180^\circ$, $\cos 180^\circ$ has the same value as $\sin 270^\circ$, and so on. In other words the graph of the cosine function is just the graph of the sine function shifted to the left $90^\circ = \pi/2$ radians, as in Figure 5.1.5.

![Figure 5.1.5 Graph of $y = \cos x$](image)

To graph the tangent function we use

$$
\begin{align*}
\tan x &= \frac{\sin x}{\cos x} \\
\tan (-x) &= \frac{\sin (-x)}{\cos (-x)} \\
\tan x &= \frac{\sin x}{\cos x}
\end{align*}
$$

to obtain the following graph

![Figure 5.1.6 Graph of $y = \tan x$](image)

### Example

Draw the graph of $y = -\tan x$ for $0 \leq x \leq 2\pi$.

Solution: Multiplying a function by $-1$ just reflects its graph around the $x$-axis. So reflecting the graph of $y = \tan x$ around the $x$-axis gives us the graph of $y = -\tan x$.
Note that this graph is the same as the graphs of \( y = \sin x \) and \( y = \cos x \) and \( y = \sin(x \pm \frac{\pi}{2}) \) for \( x \neq 0 \) and \( y = \cos(x \pm \frac{\pi}{2}) \) for \( x \neq 0 \).

**Example**

Draw the graph of \( y = 1 + \cos x \) for \( 0 \leq x \leq 2\pi \).

Solution: Adding a constant to a function just moves its graph up or down by that amount, depending on whether the constant is positive or negative, respectively. So adding 1 to \( \cos x \) moves the graph of \( y = \cos x \) upward by 1, giving us the graph of \( y = 1 + \cos x \).

**Exercise**

Draw the graph of the given function for \( 0 \leq x \leq 2\pi \).

\( y = 3 \cos x \)
Exercise
Plot the following graph using Geogebra or GNUPlot

\[ x = 2 - y \]

\[ x = 3y - 2 \]

\[ x = 6y + 4 \]

Synthesis of the Module

SYNTHESIS OF THE BASIC MATHEMATICS MODULE

We note that having gone through this module you should now be fully equipped with the concepts involved in the following contents.

In Unit 1 the most basic concepts are those of a set and function, followed by logic in which you are introduced to the science of reasoning. A good grasp of the real number system is also necessary for easy definition of elementary functions. Permutations and combinations together with trigonometric functions complete the most significant topics in this unit. These concepts are given great exposition in this unit.

In Unit 2 you have been introduced to algebraic structures in which the concept of a binary operations played a pivotal role. The concept of equivalence relation is essential. This leads to partitioning of sets into equivalence classes that facilitates deeper studies on sets or collections of spaces.

Finally in unit 3, the concepts of trigonometry are introduced. The laws and identities are presented with illustrative examples. The use of software to graph trigonometric functions is also discussed. Students will have the opportunity to manipulate graphs using GeoGebra/Gnuplot which are free.

Summative Evaluation

Module 1: Basic Mathematics

Unit 1: Summative Assessment Questions

Question 1

Write the negation of the following statement:
If I receive a salary increase I will buy a plot.

Use truth tables to show that

\[ A \lor (B \land C) \equiv (A \lor B) \land (A \lor C) \]

Determine the truth tables for the following propositions.

\[ (A \Rightarrow B) \Rightarrow (A \lor B) \]
\[ \sim (A \Rightarrow B) \lor (\sim A \land \sim B) \]

**Question 2**

Give the definition of a function. In the diagram below state with reasons whether the mapping represents a mapping or not.

Let \( A = \{ x : -2 \leq x \leq 2 \} \). Let \( f : A \rightarrow R \) and \( g : A \rightarrow R \) be defined by

\[ f(x) = 3x + 4 \]
\[ g(x) = (x - 1)^2 \]

Determine the range of each of the functions \( f \) and \( g \).

State the inverse of each of the following functions

\[ f(x) = 1 + \frac{1}{x} \]
\[ g(x) = \sqrt{3 + x^2} \]

**Question 3**

Let two functions \( f \) and \( g \) be defined on the whole set of real numbers by

\[ f(x) = x - 1 \]
\[ g(x) = 2x^2 \]

Find the composite functions (i) \( f \circ g \) and (ii) \( g \circ f \)

Given \( h(x) = x + 1 \) and \( g(x) = x^2 + 4 \) where each of these functions is defined on \( R \) find

\[ (h \circ g)^{-1} \]

The ranges of \( h \circ g \) and \( g \circ h \)

In part (b) above find the value of \( a \) such that \( (h \circ g)(a) = (g \circ h)(a) \)
**Question 4**

In how many ways can 6 boys be chosen from a class of 30 boys if the class captain is to be included?

A committee of six people is to be chosen from a group of 8 women and five men.

In how many ways can this be done?

If one particular man must not be in the team how many of these teams will have more women than men?

A box contains 15 balls, 5 of which are red, 4 are green and 6 are blue. In how many ways can three balls be chosen if

- There is no restriction?
- The balls must be of the same colour?
- Only two balls are of the same colour?

---

**Unit 1: Summative Assessment Solutions**

Q1. (a) If I do not receive a pay increase I will not buy a plot.

(b)

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(c). (i)

<table>
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<th>(A \lor B)</th>
<th>(A \rightarrow B)</th>
<th>((A \rightarrow B) \rightarrow (A \lor B))</th>
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\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
A & \sim A & B & \sim B & A \rightarrow B & \sim (A \rightarrow B) & \sim (A \land \sim B) \\
\hline
T & F & T & F & F & T & F \\
T & F & F & T & T & F & F \\
F & T & T & F & T & F & F \\
F & T & F & T & F & T & T \\
\hline
\end{array}
\]

Q2

A function is a mapping where each object is mapped onto a unique image. The diagram does not represent a function because there is an object mapping onto two different images.

Given \( A = \{x : -2 \leq x \leq 2\} \)

\[ f(x) = 3x + 4 \]

\[ g(x) = (x - 1)^2 \]

Range of \( f = \{y : -2 \leq y \leq 10\} \)

Range of \( g = \{y : 0 \leq y \leq 9\} \)

If \( f(x) = 1 + \frac{1}{x} \) then \( f^{-1}(x) = \frac{1}{x - 1} \)

If \( g(x) = \sqrt{3 + x^2} \) then \( g^{-1}(x) = \sqrt{x^2 - 3} \)

Q3

Given \( f(x) = x - 1 \)

\[ g(x) = 2x^2 \]

\[ (f \circ g)(x) = f(g(x)) = f(2x^2) = 2x^2 = 2x^2 - 1 \]

\[ (g \circ f)(x) = g(f(x)) = g(x - 1) = 2(x - 1)^2 \]

Given \( h(x) = x + 1 \)

\[ g(x) = x^2 + 4 \]

\[ (h \circ g)(x) = h(g(x)) = h(x^2 + 4) \]

\[ = x^2 + 4 + 1 \]
\[ = x^2 + 5 \]
\[ (h \circ g)^{-1}(x) = \sqrt{x - 5} \]

Range of \( h \circ g = \{y : y \geq 5\} \)

Also \( (g \circ h)(x) = g(h(x)) \)

\[ = g(x + 1) \]
\[ = (x + 1)^2 + 4 \]

Range of \( g \circ h = \{y : y \geq 4\} \)

\[ (h \circ g)(a) = a^2 + 5 \]
\[ (g \circ h)(a) = (a + 1)^2 + 4 \]

Therefore \( (a + 1)^2 + 4 = a^2 + 5 \)
\[ 2a + 5 = 5 \]
\[ 2a = 0 \]
\[ a = 0 \]

Q 4

(a). If the captain is to be included then we are selecting 5 boys from a class of 25. Hence we have

\[ ^{25}C_5 = \frac{25!}{5!20!} \]

(b) (i). This can be done in \[ ^{13}C_6 = \frac{13!}{6!7!} \]

\[ = 13 \times 12 \times 11 \]
\[ = 1716 \text{ ways} \]

We exclude the man who should not be in the committee leaving us with four men and eight women to choose from. Given that women have to be more than the men we have the following three options:

Either to select 4 women and 2 men giving us \( ^{8}C_4 \times ^{5}C_2 \) ways

Or to select 5 women and 1 man giving us \( ^{8}C_5 \times ^{4}C_1 \)

Or to select 6 women and no man giving us \( ^{8}C_6 \) ways
In total we have

\[ \binom{8}{4}x^4C_2 + \binom{8}{3}x^4C_1 + \binom{8}{2}x^2 = 420 + 224 + 28 = 672 \text{ ways} \]

(c)

\[ \binom{15}{5} \text{ ways} \]

\[ \binom{5}{4}x^4C_1x^4C_1 = 120 \text{ ways} \]

We have either 2G and 1R or 2G and 1B and 1G or 2B and 1R or 2R and 1G or 2R and 1B.

Q5

We first note that \( \sin 4\theta + \sin 2\theta = 2 \sin \theta \cos \theta \)

Therefore \( \sin 4\theta - \sin 3\theta + \sin \theta = 2 \sin 3\theta \cos \theta - \sin \theta \)

\[ = \sin 3\theta (2 \cos \theta - 1) \]

(i) \( \sec^2 x - 5(\tan x - 1) = 0 \)

\[ 1 + \tan^2 x - 5(\tan x - 1) = 0 \]

\[ \tan^2 x - 5 \tan x + 6 = 0 \]

Let \( y = \tan x \)

\[ y^2 - 5y + 6 = 0 \]

\[ (y - 3)(y - 2) = 0 \]

Therefore \( y = 3 \)

\[ y = 2 \]

\( \tan x = 3 \) or \( \tan x = 2 \)

\[ x = \tan^{-1} 3 \text{ or } x = \tan^{-1} 2 \]

(ii) \( 2(\cos^2 x - 1) + 1 - \cos^2 x = 2 \cos x \)

(c) \( \sqrt{3} \cos \theta - \sin \theta = r \cos(\theta + \alpha) \)

\[ = r \cos \theta \sin \alpha - r \sin \theta \cos \alpha \]

\[ r \sin \alpha = \sqrt{3} \text{ and } r \cos \alpha = 1 \]

\[ \therefore r = \pm 2 \text{ and } \tan \alpha = \frac{1}{\sqrt{3}} \therefore \alpha = 30^\circ \]
\[ \pm 2 \cos(\theta + 30^\circ) = 0 \]
\[ \Rightarrow \cos(\theta + 30^\circ) = 0 \]
\[ \therefore \theta + 30^\circ = 180^\circ \]
\[ \therefore \theta = 150^\circ \text{ for } 0 \leq \theta \leq 180^\circ \]

### Unit 2: Summative Assessment Questions

1. Determine whether the binary operation \(*\) on the set \(\mathbb{R}\) of real numbers is commutative or associative in each of the following cases
   
   (a) \( x * y = x^2 y \)
   
   (b) \( x * y = x y + y \)

2. (a) Let \( S \) be a non empty set with an associative binary operation \(*\) on it. For \( x, y, z \in S \) suppose that \( x \) commutes with \( y \) and \( z \). Show that \( x \) also commutes with \( y * z \).

   (b) Prove that if \( a, b \in \mathbb{Z} \) such that \( a/b \) and \( a/c \) then \( a/mb + nc \) for \( m, n \in \mathbb{Z} \).

   Where \( a/b \) means \( a \) divides \( b \).

3. Give the definition of an equivalence relation. Define a relation \( \sim \) on the set \( \mathbb{Z} \) of integers as follows: \( a \sim b \) if and only if \( a + b \) is even. Show that \( \sim \) is an equivalence relation on \( \mathbb{Z} \).

4. (a) The relation congruence modulo \( n \) on the set \( \mathbb{Z} \) of integers is defined as follows:

   For any pair \( x, y \in \mathbb{Z} \) \( x \) is said to be congruent to \( y \) modulo \( n \) written
   
   \[ xy \equiv (\text{mod} \  n) \]
   
   if \( n \) divides \( x - y \).

   Show that this is an equivalence relation.

   (b) Show that the relation \( \sim \) defined on \( \mathbb{N} \times \mathbb{N} \) by \( (a, b) \sim (c, d) \) iff \( a + d = b + c \) is an equivalence relation. Where \( \mathbb{N} \) is the set of natural numbers.
Unit 2: Summative Assessment Answers

(a) Let \( x, y, z \in \mathbb{R} \). Then \( x \ast (y \ast z) = x^2 \ast (y \ast z) = (x^2)^2 \ast (y^2 \ast z) = x^4 \ast y^2 \ast z \).

Also \( (x \ast y) \ast z = x^2 \ast y \ast z = (x^2 \ast y)^2 \ast z = x^4 \ast y^2 \ast z \).

Thus \( x \ast (y \ast z) = (x \ast y) \ast z \).

Hence \( \ast \) is associative.

We also have that:

\[ x \ast y = x^2 \ast y \text{ and } y \ast x = y^2 \ast x \]

Thus \( x \ast y \neq y \ast x \).

Hence \( \ast \) is not commutative.

(b) Let \( x, y, z \in \mathbb{R} \). Then

\[
\begin{align*}
x \ast (y \ast z) &= x \ast (yz + z) = x(yz + z) + yz + z \\
&= xyz + zx + yz + z
\end{align*}
\]

\[
(\ast) \quad (x \ast y) \ast z = (xy + y) \ast z
\]

Also

\[
= (xy + y)z + z
\]

\[
= xyz + yz + z
\]

\[
\therefore x \ast (y \ast z) \neq (x \ast y) \ast z
\]

Hence \( \ast \) is not associative.

We also have that:

\[ x \ast y = xy + y \text{ and } y \ast x = yx + x \]

\[ \therefore x \ast y \neq y \ast x \]

Hence \( \ast \) is not commutative.

(a) Given that:

\[ x \ast y = y \ast x \text{ and } x \ast z = z \ast x \]

We have by associativity of \( \ast \) that

\[
\begin{align*}
x \ast (y \ast z) &= (x \ast y) \ast z \\
&= (y \ast x) \ast z \\
&= y \ast (x \ast z) \\
&= y \ast (z \ast x) \\
&= (y \ast z) \ast x
\end{align*}
\]
Hence $x$ also commutes with $y \cdot z$.

Now $a/b \Rightarrow b = k \cdot a$ and $a/c \Rightarrow b = h \cdot a$ for some $k, h \in \mathbb{Z}$.

$$mb + nc = mka + nha = a(mk + nh)$$
$$\Rightarrow a/mb + nc$$

An equivalence relation is one which is reflexive symmetric and transitive.

Given the relation:

$$a \sim b \text{ iff } a + b \text{ is even.}$$

we have:

(i) $a + a = 2a \Rightarrow a \sim a$ reflexive

(ii) $a \sim b \Rightarrow a + b = 2k \Rightarrow b + a = 2k$

$$\Rightarrow b \sim a \text{ symmetric}$$

Let $n$ divide $x - y$. Then also $n$ divides $y - x$ which is symmetric property.

Let $n$ divide $x - y$ and also $y - z$. Then $n$ also divides $(x - y) + (y - z) = x - z$ which is transitive property. Hence congruence modulo in an equivalence relation.

We note that:

(i) $(a \sim b) \sim (a, b)$ since $a + b = b + a$ which is reflexive property.

(ii) $(a, b) \sim (c, d) \Rightarrow a + d = b + c$

$$\Rightarrow d + a = c + b$$
$$\Rightarrow (c, d) \sim (a, b)$$

which is symmetric property.
(iii) Now suppose \((a, b) \sim (c, d)\) and \((c, d) \sim (e, f)\)

Then we have that:

\[
a + d = b + c \quad \text{and} \quad c + f = d + e
\]

\[
\Rightarrow a + d + c + f = b + c + d + e
\]

\[
\Rightarrow a + f = b + e \Rightarrow (a, b) \sim (e, f)
\]

Hence \(\sim\) is an equivalence relation

---

### Unit 3: Summative Assessment Questions

#### Question 5

Simplify

\[
\sin 4\theta - \sin 3\theta + \sin 2\theta
\]

Solve for \(x\), \(0 \leq x \leq 360^\circ\)

\[
\sec^2 x - 5(\tan x - 1) = 0
\]

\[
2 \cos 2x + \sin^2 x = 2 \cos x
\]

Express \(\sqrt{3} \cos \theta - \sin \theta\) in the form \(r \cos(\theta + \alpha)\)

Hence solve for \(\theta\) in the range \(0 \leq \theta \leq 180^\circ\) the equation \(\sqrt{3} \cos \theta - \sin \theta = 0\)

2. Simplify the following expressions:

\[
\sin 3x \cos x + \sin x \cos 3x
\]

\[
2 \sin 3x \cos 3x \cos 5x - (\cos 2.3x - \sin 2.3x) \sin 5x
\]

3. Simplify \(\cos 2 \theta \tan 2 \theta\)

4. Prove that \(\frac{\tan^2 \theta}{\sec^2 \theta} = \tan^2 \theta - \sec^2 \theta\)

5. Express \(y\) in terms of \(\theta, x, \triangle\), \(\square, \triangle\), \(\square, \triangle\)
b. A laser on top of a mountain shines due north and downward on a detector at sea level. There, the laser beam makes an angle of 45° with the ground. Then the laser shines on a second detector, also due north and at sea level, which is 4200 meters north of the first detector. At the second detector, the beam makes an angle of 150° with the ground.

How far is the second detector from the laser?

Unit 3: Summative Assessment Solutions

2. 

3. Illustrating the question in the form of a diagram

5. angle a = 135°, b = 30°. Use law of sines:

\[
\frac{x}{\sin a} = \frac{4200}{\sin b}, \text{ so } x = \frac{4200 \cdot \sqrt{2}}{2} = 4200\sqrt{2}\]
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Main Author of the Module

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He has over the years participated in workshop on development of study materials for Open and Distance learning program for both science and arts students in which he has written books on Real Analysis, Topology and Measure Theory.

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